Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 08

Ph.D in Faculty of Applied Science (Mathematical Science)

ADVANCED NUMERICAL METHODS

M.Code : 77359

Time : 3 Hrs. Max. Marks : 100

INSTRUCTIONS TO CANDIDATES :

1. Attempt any FIVE questions out of EIGHT question.
2. Each question carry TWENTY marks.

1. Solve the system of equations by the Gauss-Seidal method :

   $28x + 4y - z = 32;\ x + 32y + 10z = 24;\ 2x + 17y + 4z = 35.$

2. Perform three iterations of the SOR method to find the solution of the system of equations

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

   Also, find the optimal relaxation factor and the rate of convergence.

3. Using finite difference method, solve $\frac{d^2y}{dx^2} = y$ in (0, 2), given that y(0) = 0, y(2) = 3.63.

4. Derive explicit scheme to solve the wave equation and using it to solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

   subject to u(0, t) = 0, u(1, t) = 0 for t > 0 and $\frac{\partial u}{\partial t}(x, 0) = 0$, $u(x, 0) = \sin 3\pi x$ for all $x \in [0,1]$.

5. Using Galerkin method, solve the equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 2y = 0, y'(0) - y(0) = -1$ and y(1) = 2

6. Consider the boundary value problem $u''' - (1 - x^2)u' - 1 = 0, u(-1) = 0$. Use the Galerkin method determine the coefficients of the approximate solution

$$w(x) = u_0(1 - x^2)(1 - 4x^2) + \frac{16}{3}u_1(x^2 - x^4)$$

Where u0,u1 are the unknown solution values at the nodes 0 and ½ respectively.

7. Using finite element method, solve

$$\frac{d^2y}{dx^2} - 2 = 0, 0 < x < 1, \; y(0) = 0, y'(1) = 0.$$

8. Solve by the finite element method $y'' - xy' - x^3 = \frac{4}{x^3}$, $y(1) = -1, y(2) = 3$. Put nodes at x = 1.2, 1.5 and 1.75 as well as at the ends of [1, 2], Compare your solution to the analytical solution given by $y = x^2 - \frac{2}{x}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.