Roll No. | | | | | | | | | | | | | Total No. of Pages : 03

Total No. of Questions : 08

Ph.D in Faculty of Applied Science (Mathematical Sciences)

LINEAR ALGEBRA

M.Code : 77373

Time : 3 Hrs. Max. Marks : 100

INSTRUCTIONS TO CANDIDATES :

1. **Attempt any FIVE questions out of EIGHT question.**
2. **Each question carry TWENTY marks.**

1. a) Solve the following system of equations using gauss elimination method

$-12x + y + 2z = 2$

$3x - 12y + z = 6$

$-2x + 3y + 12z = 4$

b) Show that columns of $\begin{bmatrix} 2 & 7 & 5 \\ 3 & -6 & 2 \\ 1 & 17 & 7 \end{bmatrix}$ are linearly dependent.

2. a) State and prove rank nullity theorem.

b) Prove that $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{bmatrix} = xy$

3. a) Find the fourier transform of e^{-ax^2}, where $a > 0$

b) Fit by a straight line

(0,3), (2,1) (3,–1), (5,–2)

4. a) Transform the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ to tridiagonal form by Given's method.

Find the eigen vector corresponding to largest eigen value.

b) Find the linear transformation T: $R^3 \to R^4$ whose range is spanned by (1,2,0,–4) and (2,0,–1,–3)

5. a) Test the definiteness of the following function :

$$x_1^2 + 4x_2^2 + 4x_3^2 + 4x_1x_3 + 16x_2x_3$$

b) Prove that a linear system of equations Ax = b whose condition number is small is well conditioned.

6. a) Use simplex method to solve :

Max $Z = 2x_1 + 3x_2$

Subject to : $x_1 + x_2 \leq 4$

$-x_1 + x_2 \leq 1$

$x_1 + 2x_2 \leq 5$

$x_1, x_2 \geq 0$

b) Use Dual Simplex Method to solve the LPP :

Minimize $Z = 3x_1 + x_2$

Subject to : $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

$x_1 , x_2 \geq 0$

7. a) Solve the game graphically

$$\begin{array}{c} \\ Player\ A \end{array} \begin{array}{c} \begin{array}{cc} Player\ B & \end{array} \\ \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \end{array}$$

b) Solve the following using dominance property :

$$\begin{array}{c} \\ Player\ A \end{array} \begin{array}{c} \begin{array}{cccc} Player\ B & & & \end{array} \\ \begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{array}$$

8. Determine the Maximum flow and the optimum flow in each arc for the following network.

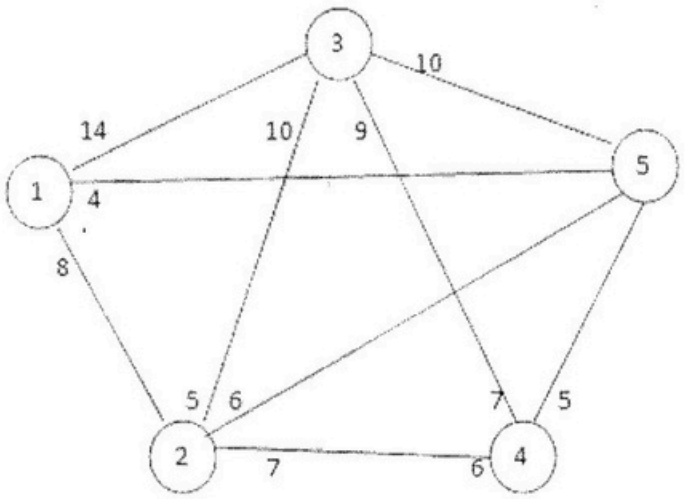

Fig.1

Also determine :

a) The surplus capacities for all arcs.

b) Amount of flow through nodes 2, 3, and 4.

c) Can the flow be increased by increasing the capacities in the arcs 3-5 and 4-5.

d) Does the problem have alternate solutions? If yes, find it.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.