

Roll No.

Total No. of Pages : 03

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Ph.D Examination
LINEAR ALGEBRA (Mathematics)

M.Code : 78005

Time : 3 Hrs.

Max. Marks : 100

INSTRUCTIONS TO CANDIDATES :

1. Attempt any FIVE questions out of EIGHT questions.
2. Each question carries TWENTY marks.

1. a) Solve the following system of equations using gauss Jordan method :

$$-x + y + 2z = 2$$

$$3x - y + z = 6$$

$$-x + 3y + 4z = 4$$

- b) Give examples of two different basis of $V_2(\mathbb{R})$ and $V_3(\mathbb{R})$.

2. a) State and prove rank nullity theorem.

- b) Solve the following using determinants :

$$x + y + z = 1$$

$$ax + by + cz = k$$

$$a^2x + b^2y + c^2z = k^2$$

3. a) Find the fourier transform of $f(x) = k$ if $0 < x < a$, and $f(x) = 0$

- b) Fit by a straight line

$(0,3), (2,1), (3,-1), (5,-2)$

4. a) Transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to tridiagonal form by Given's method. Find the eigen vector corresponding to largest eigen value.

- b) Find the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ whose range is spanned by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$

5. a) Test the definiteness of the following function :

$$x_1^2 + x_2^2 + x_3^2 - 2x_1x_2$$

- b) Prove that a linear system of equations $Ax = b$ whose condition number is small is well conditioned.

6. a) Use Big M method to Maximize $Z = 6x_1 + 4x_2$

$$\text{Subject to : } 2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1, x_2 \geq 0$$

- b) Use Dual Simplex Method to solve the LPP :

$$\text{Minimize } Z = 3x_1 + x_2$$

$$\text{Subject to : } x_1 + x_2 \geq 1$$

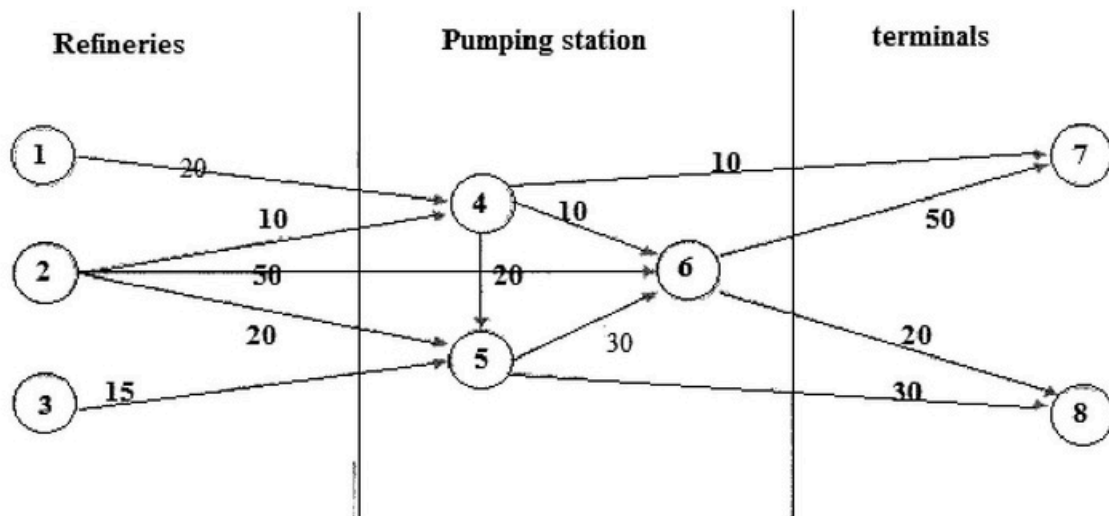
$$2x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

7. Solve the following game by linear programming using simplex method and determine the best strategies for both the players.

		Player B		
PlayerA	3	-4	2	
	1	-3	-7	
	-2	4	7	

8. Three refineries send LPG to two distribution terminals through a pipeline network. Any demand that cannot be satisfied through the network is acquired from other source. The pipeline network is served by 3 pumping stations as shown in the figure. The LPG flows in the network in the direction of arrows. The capacity of each pipeline is mentioned on the arcs in [million bbl per day]. To match the maximum capacity of network, determine the following :
- Daily production at each refinery
 - The daily demand at each terminal
 - The daily capacity of each pump.



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