Roll No. **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. (Mathematics) (2018 Batch) (Sem.-3)

TOPOLOGY

Subject Code : MSM-301-18

M.Code : 76672

Date of Examination : 12-12-2022

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. Write short notes on :

a) Define a locally connected space.

b) What is meant by a sub-basis and basis for a topological space. Give examples.

c) What is a regular space.

d) State Urysohn's Lemma.

e) Give one example of a T_2 space.

SECTION-B

2. a) Give example of two basis for the usual topology on the plane $\mathbb{R}^2$. State and prove a necessary and sufficient condition for two bases to give the same topology.

b) Prove or disprove : Each component of a locally connected space is open.

3. a) When is a function sequentially continuous at a point? Let $f:\mathbb{R}\to\mathbb{R}$ continuous. If $Y\subset\mathbb{R}$ is an interval, show that $f(Y)$ is connected, state and prove one of the main results used.

   b) Let U be an open subset of a topological space X and S be dense in X. Prove $\overline{S\cap U}=\overline{U}$.

4. a) What do you mean by compactification of a space? Prove that union of two compact subsets of a space is compact.

   b) Prove that a sequence x_n in $\mathbb{R}$ (with usual topology) converges if and only if, for every pair of positive integers n and m, there is a positive integer N such that $|x_n - x| < \frac{1}{n}\ \forall\ n,m > N$

SECTION-C

5. a) Show that for a Hausdorff topological space to have a Hausdorff one point compactification, it is necessary and sufficient that it is locally compact and non-compact.

   b) Let X be a T_2 space. Show that if a point x of x is a limit point of a subset A of X, then every neighbourhood of x contains infinitely many elements of A.

6. a) State and Prove Tietze Extension theorem.

   b) Prove that every compact subset of a Hausdorff space is closed.

7. a) Show that a sequentially compact topological space is countably compact. Is the converse true? Justify your answer.

   b) Prove that every metric space is normal.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.