Roll No. | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. (Mathematics) (2018 Batch) (Sem.-3)

MECHANICS-II

Subject Code : MSM-305-18

M.Code : 76676

Dateof Examination : 16-12-22

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. All questions are compulsory.

   a) What is dummy suffix in the summation notation?

   b) Prove : $\delta_{ij}\, a_j = a_i$

   c) What is tensor function?

   d) If A is a symmetric tensor with components a_{ij} and B is a skew tensor with components b_{ij}, then show that A . B = $a_{ij}b_{ij} = 0$.

   e) What is deformation?

SECTION-B

2. a) Given the matrix $[a_{ij}]$, consider the matrix $[a_{ij}^*]$ where $a_{ij}^* = \frac{1}{2}\varepsilon_{ipq}\varepsilon_{jrs}a_{pr}a_{qs}$. Show that $[a_{ij}][a_{ij}^*]^T = [a_{ij}^*]^T[a_{ij}] = D[I]$, where D = det ($a_{ij}$). Deduce that, if D $\neq$ 0, $[a_{ij}]^{-1} = \frac{1}{D}[a_{ji}^*]$.

   b) Find the spherical and deviatoric parts of the tensor A whose matrix is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

c) Prove that the principal directions corresponding to distinct principal values of A are orthogonal.

3. a) If $a_{ij} = a_{ji}$ and $b_{ij} = -b_{ji},$ then prove that $a_{ij}b_{ij} = 0$.

   b) Prove that the representation of a second-order tensor as a sum of a symmetric tensor and a skew tensor is unique.

4. a) If a_{ij} are components of an isotropic tensor (of second order), then prove that $a_{ij} = \alpha\delta_{ij}$ for some scalar α.

   b) If the corresponding components of two tensors of the same order are equal in one coordinate system, then show that they are equal in all coordinate systems.

SECTION-C

5. a) Let C be a simple closed curve in three-dimensional space and S be an open regular surface bounded by C. Then for a vector field define on S as well as C, $\oint_C At\, dS = \int_S (curl\ A)^T n dS$, where t is the unit tangent to C, which is assumed to be positively oriented relative to the unit normal n to S.

   b) For the deformation defined by equations : $x_1 = x_1^0 + x_2^0$, $x_2 = x_1^0 - 2x_2^0$, $x_3 = x_1^0 + x_2^0 - x_3^0$, find F, F^{-1} and J.

6. a) Show that $\nabla^2 |x|^n = 0$ only if $n = 0$ or -1.

   b) For small deformation, show that the normal strain of a material arc assumes an extremum value when the elements lays along a principal direction of strain.

7. a) If c is a constant vector, show that $f(r)\, c \times x$ is divergence free.

   b) If e_{13} and e_{23} are the only non-zero strain components and e_{13} and e_{23} are independent of x_3, show that the compatibility conditions may be reduced to the single condition : $e_{13.2} - e_{23.1}$ = constant.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.