Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc. (Mathematics) (Sem.-2)

REAL ANALYSIS-II

Subject Code : MSM-202-18

M.Code : 75963

Date of Examination : 23-12-22

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. **Write short notes on :**

   a) State inverse function theorem.

   b) Define Lebesgue outer measure of a set.

   c) If E is a measurable set, then E^c is also measurable.

   d) Define Lebesgue integral.

   e) Define functions of bounded variations.

SECTION-B

2. a) Define the derivative of a real function. Let f be a function defined on $[a, b]$. If f is a differentiable at a point $x \in [a, b]$, then f is continuous at x.

   b) Let f be a function defined by $f(x)=\begin{Bmatrix} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{Bmatrix}$. Is this function differentiable at $x = 0$? Discuss your answer in detail.

3. a) If E_1 and E_2 are measurable sets, then $E_1 \cup E_2$ is measurable.

b) If A and B are disjoint subsets of $\mathbb{R}$, then prove that $m^*(A \cup B) = m^*(A) + m^*(B)$, where m^* denotes the Lebesgue outer measure.

4. a) Define Lebesgue measurable function. Show that the function ϕ on $\mathbb{R}$ defined by

$$\phi(x) = \begin{Bmatrix} x+5, & if & x < -1 \\ 2, & if & -1 \le x < 0 \\ x^2, & if & x \ge 0 \end{Bmatrix}$$

Is a Lebesgue measurable function.

b) State and prove Taylor's theorem.

SECTION-C

5. a) Let f and g be bounded measurable functions defined on a set E of finite measure, then

i) $\int_E (f+g) = \int_E f + \int_E g$, ii) $a\int_E f$, for all real number a.

b) Give an example of a Lebesgue integrable function which is not Riemann integrable.

6. a) If f is absolutely continuous function on $[a,b]$ then f is a function of bounded variation on $[a, b]$.

b) If f is a measurable function over the interval $[a, b]$, then the function f is Lebesgue integrable over $[a, b]$ if and only if $|f|$ is Lebesgue integrable over $[a, b]$.

7. State the prove Lebesgue differentiation theorem. Also verify Lebesgue differentiation theorem for the function $f(x) = \begin{cases} 0, & if\ 0 \le x \le 1 \\ 1, & if\ 1 < x \le 2 \end{cases}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.