Roll No.

Total No. of Questions: 07

M.Sc. (Mathematics) (Sem. - 2)

PARTIAL DIFFERENTIAL EQUATIONS

Subject Code: MSM-204-18

M Code: 75965

Date of Examination : 19-12-2022

Time: 3 Hrs.

Max. Marks: 70

Total No. of Pages: 02

INSTRUCTIONS TO CANDIDATES:

- 1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
- 2. SECTION-B contains THREE questions carrying FIFTEEN marks each and students have to attempt any TWO questions.
- 3. SECTION-C contains THREE questions carrying FIFTEEN marks each and students have to attempt any TWO questions.

SECTON-A

- 1. Write briefly:
 - a) Construct a partial differential equation of all spheres whose centres lie on the z-axis, by the elimination of arbitrary constants.
 - b) Define order of a partial differential equation with an example.
 - c) Write two dimensional wave equation.
 - d) What are the assumptions made for deriving the equation of a vibrating string?
 - e) Solve the linear partial differential equation $\frac{\partial^4 z}{\partial x^4} \frac{\partial^4 z}{\partial y^4} = 0.$

SECTION-B

- 2. a) Solve the partial differential equation pxy + pq + qy = yz, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ by Charpit's method.
 - b) Solve $z^2(p^2 + q^2) = x^2 + y^2$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

- 3. a) Solve $\frac{\partial^2 z}{\partial x^2} \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$.
 - b) Solve $\frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} \cos 2x + \frac{\partial f}{\partial x} \tan x = 0$, by Monge's method.

4. a) Solve
$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$
, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

b) Solve $z^2(p^2x^2 + q^2) = 1$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

SECTION-C

5. a) Using the method of separation of variables, solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$.

b) A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y = y_0 \sin^3 \left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement y(x, t).

- 6. Find the deflection u(x, y, t) of the square membrane with a = b = c = 1, if the initial velocity is zero and the initial deflection $f(x, y) = A \sin \pi x \sin 2\pi y$.
- 7. Solve $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions
 - a) $u \neq \infty$ if $t \rightarrow \infty$;
 - b) $u(0,t) = u(\pi,t) = 0;$
 - c) $u(x,0) = \pi x x^2$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.