Roll No. | | | | | | | | | | | | | **Total No. of Pages: 02**

Total No. of Questions: 07

M.Sc. (Mathematics) (Sem. – 2)

MECHANICS-I

Subject Code: MSM-203-18

M Code: 75964

Date of Examination: 16-12-2022

Time: 3 Hrs. **Max. Marks: 70**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION-B contains THREE questions carrying FIFTEEN marks each and students have to attempt any TWO questions.**
3. **SECTION-C contains THREE questions carrying FIFTEEN marks each and students have to attempt any TWO questions.**

SECTON-A

1. Write briefly:

a) Use Euler's equation to find the extremum of the integral $J = \int_0^{\pi/2} (y'^2 - y^2)\mathrm{d}x$, where $y(0) = 0, y(\pi/2) = 1$.

b) Give an example of holonomic and non-holonomic constraint.

c) What are the generalized co-ordinates and degree of freedom of particle moving on inside surface of a cone?

d) Show that the total kinetic energy of a system of particles can be expressed as the sum of kinetic energy of motion of centre of mass and the kinetic energy of motion about the center of mass.

e) If $[\Phi, \Psi]$ is the Poisson bracket of Φ and Ψ, then prove that

$$\frac{\partial}{\partial t}[\Phi, \Psi] = \left[\frac{\partial \Phi}{\partial t}, \Psi\right] + \left[\Phi, \frac{\partial \Psi}{\partial t}\right]$$

SECTION-B

2. a) Use variational principle and derive the Euler's equation for one dependent function of one variable.

b) Find the minimum surface of revolution formed by revolving a curve about the x-axis?

3. a) Write the equations of motion for a particle of mass, m moving under central force, following the inverse square law, using Lagrange's equations of motion.

   b) State D'Alembert's principle and deduce Lagrange's equation for conservative system using this.

4. a) If the two end-points of a line segment of length l are moving along two orthogonal straight line, then find the locus of the point on the line segment which divides in the ratio of 1:2.

   b) Show using Lagrange equations that if a given component of total applied force vanishes, the corresponding component of linear momentum is conserved.

SECTION-C

5. a) State and prove the principle of least action.

   b) Show that if time, t is a cyclic co-ordinate, then Hamiltonian is the constant of motion.

6. a) Prove the Jacobi-Poisson's theorem and using this show that the Poisson's bracket of two constants of motion is itself a constant of motion.

   b) If the transformations equations between two sets of co-ordinates are

$$P = 2\left(1 + q^{1/2}\cos(p)\right)q^{1/2}\sin(p)$$
$$Q = \log\left(1 + q^{1/2}\cos(p)\right)$$

   then show that the transformation is canonical.

7. a) A particle of mass m, is constrainted to move on the surface of a cylinder. The particle is subjected to a force directed towards the origin and proportional to the distance of the particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion.

   b) Show the $\sum_{l=1}^{2n}\{u_l, u_i\}\left[u_l, u_j\right] = \delta_{ij}$, where $\{u_l, u_i\}$ is Lagrange bracket, while $\left[u_l, u_j\right]$ is Poisson bracket.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.