Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

M.Sc. (Mathematics) (Sem.–1)

REAL ANALYSIS-I

Subject Code : MSM-102-22

M.Code : 92797

Date of Examination : 17-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select at least TWO questions from SECTION - B & C.**

SECTION-A

1. Write briefly :

a) Give example of a countable set and an uncountable set.

b) Define Pseudo metric space.

c) Give an example of metric space.

d) Define Compact set.

e) Define convergence of a sequence in a metric space.

f) Define complete metric space.

g) Define continuous function in a metric space.

h) Discuss different kinds of discontinuity.

i) Define partition of a closed interval [a, b].

j) Define upper and lower Riemann-Stieltjes integrals.

SECTION-B

2. a) Show that the set of rational numbers is countable.

   b) Show that in a metric space, arbitrary union of open sets is open.

3. a) Show that in the usual metric space (R,d)

   i) (2, 3) and (4, 5) are separated sets (ii) (2, 3) and (2,5) are not separated sets,

   b) Let d be a metric on ^{2}, describe open spheres of unit radius centered at O(0,0) when $x = (x_1, x_2) \in \mathbb{R}^2$ and $y = (y_1, y_2) \in \mathbb{R}^2$, d is given by

   $$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} .$$

4. a) Show that limit of a sequence in a metric space, if it exists is unique.

   b) Show that every convergent sequence in a metric space is a Cauchy sequence.

5. Suppose that $X_n \in \mathbb{R}^2$ (n = 1, 2, 3,...) and $X_n = (\alpha_{1n}, \alpha_{2n}, \ldots, \alpha_{kn})$. Then the sequence $\{X_n\}$ in $\mathbb{R}^k$ converges to the point $X = ((\alpha_1, \alpha_2, \ldots, \alpha_k)$ in $\mathbb{R}^k$ if and only if $\lim_{n\to\infty} \alpha_{jn} = \alpha_j$; $(1 \le j \le k)$.

SECTION-C

6. Let (X, d_1) and (Y, d_2) be two metric spaces and let $f : X \to Y$ be a mapping. Then f is continuous if and only if inverse image under f of every open subset of Y is an open subset of X.

7. Define uniform continuity in a metric space. Show that every uniform continuous function is continuous but converse of this need not be true.

8. Let $f : [a, b] \to \mathbb{R}$ be bounded and $\alpha : [a, b] \to \mathbb{R}$ be a monotonically increasing, then for any partition P of $[a, b]$, $m(\alpha(b) - \alpha(a)) \le L(P, f, \alpha) \le U(P, f, \alpha) \le M(\alpha(b) - \alpha(a)$ where m and M are g.l.b and l.u.b of f in $[a, b]$.

9. If $f_1 \in R(\alpha)$ and $f_2 \in R(\alpha)$ on $[a,b]$, then $f_1 + f_2 \in R(\alpha)$ and

   $$\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$$

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.