Roll No. [ ] Total No. of Pages : 02

Total No. of Questions : 07

M.Sc. (Mathematics) (Sem.-1)

REAL ANALYSIS-I

Subject Code : MSM-102-18

M.Code : 75130

Date of Examination : 10-01-2023

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. **Solve the following:**

a) Define cantor set and shows that it is uncountable.

b) Evaluate $\int_0^3 x^2 d([x]-x)$.

c) Show that every infinite subset of a countable set is countable.

d) Let (X, d) be a metric space. How many more metrices can we define on X? Explain.

e) Define complete metric space.

SECTION-B

2. a) Prove that closed subset of a compact set is compact.

b) Let Y be subspace of a metric space (X, d). Then

$E \subseteq Y$ is open in Y(=) $E = G \cap Y$, G is open set in X.

3. a) Show that in real numbers intervals & only intervals are connected sets.

   b) State & prove Max-Min theorem.

4. a) Prove that every Cauchy Sequence R^K is convergent.

   b) Give an example to show that a continuous function defined on a non-compact domain doesn't attain its bounds.

SECTION-C

5. a) If $f \in R(\alpha)$ on $[a, b]$ and $a < c < b$ then show that $f \in R(\alpha)$ on $[a,b]$ & $[c, b]$.

   b) Check uniform convergence of series $\sum_{n=0}^{\infty} xe^{-nx}$ on [0, 1]

6. State & prove Weierstrass approximation theorem.

7. a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

   b) If $f \in R(\alpha)$ on $[a, b]$ & c be any constant then $cf \in R(\alpha)$ on $[a, b]$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.