Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 11

M.Sc. (Mathematics) (2018 Batch) (Sem.–1)

REAL ANALYSIS-I

Subject Code : MSM-102-18

M.Code : 75130

Time : 3 Hrs. Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
2. SECTION - B & C. have THREE questions each.
3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
4. Select atleast TWO questions from SECTION - B & C each.

SECTION-A

1. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n}{n^2} x^n$.

2. State Weierstrass M-test for uniform convergence of sequence of functions.

3. State Cauchy’s principle of uniform convergence for sequences of functions.

4. State Abel's test for uniform convergence.

5. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a, b] then prove that $fg \in R(\alpha)$

SECTION-B

6. State and prove Heine-Borel theorem.

7. a) State and prove Dirichlet’s theorem on power series.

   b) Prove that the power series $\sum_{n=0}^{\infty} a_n x^n$ and the corresponding series of derivatives $\sum_{n=0}^{\infty} n a_n x^{n-1}$ have the same radius of convergence.

8. a) Show that for $0 < x < \pi / 2$, $\frac{\tan x}{x} > \frac{x}{\sin x}$.

b) Prove that the set of real numbers in [0, 1] is uncountable.

SECTION-C

9. a) Let α be monotonically increasing function on [a, b] and $f_n \in R(\alpha)$ on [a, b], for n = 1, 2, 3, ..., such that $f_n \to f$ uniformly on [a, b]. Then $f \in R(\alpha)$ on [a, b] and

$$\int_a^b f\,d\alpha = \lim_{n\to\infty} \int_a^b f_n\,d\alpha.$$

b) If f is monotonic on [a, b], and if α is continuous on [a, b], then $f \in R(\alpha)$.

10. a) Let $\{a_n\}$ be a decreasing sequence of positive terms. Prove that the ser $n \sin x$ converges uniformly on R if. and only if $na_n \to 0$ as $n \to \infty$.

b) Suppose that the sequence $\{f_n\}$ of functions converges uniformly to f on a set E in a metric space X. Let x be the limit point of E and such that $\lim_{t\to x} f_n(t) = A_n$ (n = 1, 2, 3, ...). Then $\{A_n\}$ converges and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$.

11. State and prove Stone Weierstrass theorem.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.