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M.Sc Mathematics (PIT) (2019 Batch) (Sem.–1)
COMPLEX ANALYSIS
Subject Code : UC-MSM-103-18
M.Code : 77280

Time : 3 Hrs.   Max. Marks : 60
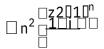
INSTRUCTIONS TO CANDIDATES :
1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. What is radius of convergence, find radius of convergence of $\sum (3-4i)^n z^n$.

2. Explain singularities of a analytic function, and give classification of them.

3. Show that the function $u = \cos x \cosh y$ harmonic and find its harmonic conjugate.

4. Determine the regular function whose imaginary part is

$$e^{-x} (x \cos y + \sin y)$$

5. Find the domain of convergence of the series

$$\sum n^2 \left[\frac{z-1}{1+i} \right]^n$$

6. Solve the equation $\sinh z = i$.

7. Use Rouche's theorem to determine the number of roots of the equation

$$z^8 - 4z^5 + z^2 - 1 = 0$$

8. Find the bilinear transformation which maps $z = 1, i, -1$ respectively onto $w = i, 0, - i$.

9. Define Bilinear transformation, Critical point, Fixed point.

10. Find the transformation which maps outside $|z| = 1$, on the half plane $\text{Re}(w) \geq 0$ so that the points $z = 1, -i, -1$ correspond to $w = i, 0, -i$.

SECTION-B

11. Show that the function $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

Also determine the corresponding analytic function $f(z)$ in terms of z.

12. a) Prove that a circle on the sphere corresponds to a circle or a straight line in z-plane.

b) Describe the curve on the sphere to which any straight line on the z-plane correspond.

13. State and prove Fundamental theorem of algebra.

14. a) State and prove Maximum Modulus Principle.

b) State and prove Liouville's theorem.

SECTION-C

15. Prove that $\int\limits_{0}^{\infty} \dfrac{\sin x}{x(x^2 + a^2)}\, dx = \dfrac{\pi}{2a^2}(1 - e^{-a}),\ a \geq 0.$

16. Find the residue of $f(z)$ at $z = 1, 2, 3$

$$f(z) = \frac{z^3}{(z-1)^4\,(z-2)(z-3)}$$

17. Find the bilinear transformation which transforms the half plane $\text{Re}(z) \geq 0$ into the unit circle $|w| \leq 1$.

18. Find the general bilinear transformation which transform the circular disc. $|z| \leq \rho$ onto the circular disc $|w| \leq \rho'$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.