Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

Master of Science (Mathematics) (Sem.–1)

ORDINARY DIFFERENTIAL EQUATIONS AND SPECIAL FUNCTIONS

Subject Code : MSM-104-22

M.Code : 92799

Date of Examination : 21-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Write short notes on :

a) State Lipschitz condition.

b) State Picard's existence and uniqueness theorem.

c) Define initial value and boundary value problems of ordinary differential equations.

d) Convert $y'' - 4y' + 4y = \sin x$, $y(0) = 1, f'(0) = 2$ to a system of two first order ODEs.

e) State Sturm-Liouville problem of ordinary differential equations.

f) What do you mean by ordinary points?

g) State Legendre's differential equation.

h) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

i) Show that $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2-2x}}$.

j) State Chebyshev's differential equation.

SECTION-B

2. a) Solve the differential equation $y'' - 6y' + 9y = \sin x$.

   b) Apply Picard's method to solve the initial value problem up to third approximation $\frac{dy}{dx} = 3e^x + 2y;\ y(0) = 0$

3. a) Show that the function $f(x, y) = y^{\frac{2}{3}}$ does not satisfy the Lipschitz condition on the rectangle R : $|x| \le 1, |y| \le 1$.

   b) Solve the following system of equations using operator method:

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$$

4. Find the Eigen values and the corresponding Eigen functions of

$$X'' + \lambda X = 0,\ X(0) = 0,\ X'(L) = 0$$

5. Obtain the formal expansion of the function $f(x) = \pi x - x^2$, $0 \le x \le \pi$ the series of orthonormal characteristic functions of Sturm-Liouville problem $y'' + \lambda y = 0$, $y(0) = y(\pi) = 0$.

SECTION-C

6. Solve in series $\frac{d^2 y}{dx^2} + xy = 0$.

7. What do you mean by orthogonality of Bessel functions.

8. Prove that $e^{2tx - t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$

9. Show that $L_{n-1}^{k}(x) + L_n^{k-1}(x) = L_n^{k(x)}$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.