Roll No. | | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc Mathematics (2018 Batch) (Sem.–1)

ORDINARY DIFFERENTIAL EQUATIONS AND SPECIAL FUNCTIONS

Subject Code : MSM-104-18

M.Code : 75132

Date of Examination : 12-01-2023

Time : 3 Hrs. **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. Answer briefly :

a) Solve the differential equation $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 10y = 0$.

b) State existence and uniqueness theorem for solution system of linear differential equations.

c) Find the solution of $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^2y = 0$ in terms of Bessel's function.

d) Write down the expression for Laguerre's polynomial.

e) Evaluate the following integral in terms of Bessel's function : $\int x^2 J_1(x)\, dx$

SECTION –B

2. Use the operator method to find general solution of the following linear system $\frac{dx}{dt}+\frac{dy}{dt}-x-3y=3t,\ \frac{dx}{dt}+2\frac{dy}{dt}-2x-3y=1$.

3. Employ Matrix method (using Eigen values of Eigen vectors) to find the general solution of the following homogeneous linear system by $\frac{dx}{dt}=x-y-z$, $\frac{dy}{dt}=x+3y+z$, $\frac{dz}{dt}=-3x-6y+6z$.

4. Find the characteristics values and characteristic functions of the following Strum-Liouville problem $\frac{d^2y}{dx^2}+\lambda y=0$, $y(0)=0, y'(\pi)=0$.

SECTION-C

5. Find the series solution of $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-(x^2+\frac{5}{4})y=0$.

6. a) Prove that $J_n'(x)=\frac{1}{2}[J_{n-1}(x)-J_{n-1}(x)]$.

   b) Express x^2-x+1 in terms of Langendre polynomials.

7. Sow that Hermite polynomials are orthogonal over $(-\infty, \infty)$ with respect to the weight function e^{-x^2}.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.