Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 09

M.Sc. (Mathematics). (Sem.–1)

MATHEMATICAL METHODS

Subject Code : MSM-105-22

M.Code : 92800

Date of Examination : 23-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Write short notes on :

a) Define Laplace transform.

b) Find the Laplace transform of sin *at*.

c) What are sufficient conditions for existence of Laplace transform of a function?

d) Define Fourier transform.

e) What are applications of Fourier transform?

f) State convolution theorem of Fourier transform.

g) Give examples of Fredholm and Volterra integral equations.

h) Give two names of methods for solving integral equations.

i) Give an example of an integral equation with separable kernel.

j) Define symmetric kernel of an integral equation. Also give an example.

SECTION-B

2. a) State and prove linearity property of Laplace transform.

   b) Find the Laplace transform of (i) $\cos^2 2t$ (ii) $\sin^3 2t$.

3. a) Solve the following equation using Laplace transform

   $y'' - 2y' + y = e^t$ with $y(0) = 2$, $y'(0) = -1$.

   b) Find the inverse Laplace transform of $\dfrac{s+2}{s^2 - 4s + 13}$.

4. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$.

5. Solve the following equation using Fourier sine transform $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ subject to the following conditions.

   a) $u = 0$ when $x = 0$, $t > 0$

   b) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \le 1 \text{ when } t = 0 \end{cases}$

   c) $u(x, t)$ is bounded.

SECTION-C

6. a) Convert the differential equation $y''(x) - 2xy'(x) - 3y(x) = 0$, $y(0) = 1$, $y'(0) = 0$ into an integral equation.

   b) Show that $y(x) = 2 - x$ is a solution of the integral equation $\int_0^x e^{x-t} y(t)\, dt = e^x + x - 1$.

7. Using the method of successive approximations, solve the following integral equation $y(x) = 1 + \lambda \int_0^1 xt\, y(t)\, dt$.

8. Find the eigenvalues and eigenfunctions of the following homogeneous integral equation with degenerate kernel $y(x) = \lambda \int_0^1 (3x - 2)t\, y(t)\, dt$.

9. Using Fredholm determinant, find the resolvent kernel of the integral equation $y(x) = f(x) + \lambda \int_0^1 xe^t\, y(t)\, dt, (\lambda \neq 1)$ and hence solve it.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.