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Total No. of Questions : 09

M.Sc. (Mathmatics) (Sem.–1)

COMPLEX ANALYSIS

Subject Code : MSM-103-22

M.Code : 92798

Date of Examination : 19-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Write short notes on :

a) Show that real and imaginary parts of a complex number is satisfy the Laplace equation.

b) Construct the analytic function $f(z) = u + iv$, where $u = x^3 - 3xy^2 + 3x + 1$.

c) Using the definition of an integral as the limit of a sum evaluate $\int_L dz$, where L is any rectifiable arc joining the points $z = \alpha$ and $z = \beta$.

d) State Cauchy's integral formula.

e) If C is a circle $|z - 2| = 5$, determine whether $\int_C \frac{dz}{z-3}$ is zero.

f) Expand the function $f(z) = \sin z$ in a Taylor's series about $z = 0$.

g) State Laurent's theorem for a function of two variables.

h) Define Poles and Isolated essential singularity with the help of an example.

i) Define conformal mapping.

j) Consider the transformation $w = T_1(z) = \frac{z+2}{z+3}$ and $w = T_2(z) = \frac{z}{z+1}$ find $T_2^{-1}\, T_1(z)$.

SECTION-B

2. Show that an analytic function cannot have a constant absolute value without reducing to a constant.

3. If $f(z)=\dfrac{x^2 y\,(y-ix)}{x^6+y^2}\,(z\neq 0), f(0)=0$, prove that $\dfrac{f(z)-f(0)}{z}\to$ *as* $z\to 0$ along any radius vector but not as $z\to 0$ in any manner.

4. State and prove Cauchy-Goursat theorem.

5. Let $f(z)$ be analytic within and on the boundary C of a simply connected region D and let z_0 be any point within C, then

$$f'(z)=\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^2}\,dz$$

SECTION-C

6. Expand $\dfrac{1}{(z+1)\,(z+3)}$ in a Laurent's series valid for the regions

a) $|z|<1$ b) $1<|z|<3$ c) $|z|>3$ d) $1<|z+1|<2$

7. Show that $\displaystyle\int_0^{2\pi}\frac{d\theta}{a+b\cos\theta}=\frac{2\pi}{\sqrt{a^2-b^2}}, a>b>0.$

8. Consider the transformation $w=e^{i\pi/4}$ and determine the region in the w-plane corresponding to the triangular region bounded by the lines $x=0$, $y=0$ and $x+y=1$ in the z-plane.

9. Find a bilinear transformation, which transforms $|z|=1$ into the real axis in such a way that the point $z_1=1$ is mapped into the points $w_1=0$, the point $z_2=-1$ is mapped into $w_2=1$ and the point $z_3=-1$ is mapped into $w_3=\infty$. Into what regions the interior and exterior of the circle mapped.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.