Roll No.						

Total No. of Pages : 02

Total No. of Questions : 07

M.Sc. (Mathematics) (Sem.-1) COMPLEX ANALYSIS Subject Code : MSM-103-18 M.Code : 75131 Date of Examination : 19-01-2023

Time: 3 Hrs.

Max. Marks : 70

INSTRUCTIONS TO CANDIDATES :

- 1. SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.
- 2. SECTION B & C have THREE questions each.
- 3. Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.
- 4. Select atleast TWO questions from SECTION B & C each.

SECTION-A

1. Write short answers of the following :

- a) Prove that ii is wholly real.
- b) Find the length of the arc C : $z(t) = (1 i) e it, 0 \le t \le \frac{\pi}{2}$
- c) State Poisson Integral formula.
- d) Write the series expansion of sinh z.
- e) Write a short note on transformations.

SECTION-B

2. a) Show that the function $\frac{1}{z^4}$, $z \neq 0$, is analytic in the given domain and determine f'(z).

b) Prove that
$$\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$$
. Hence, evaluate. $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$

- 3. Verify that the maximum and minimum modulus theorems hold for the functions
 - a) $f(z) = e^{z}$
 - b) $f(z) = z^2 + 1$ where D is the domain $|z| \le 1$.

4. a) Find the upper bound for the absolute value of the $I = \int_C \frac{(z^2 + 3) e^{iz} Log z}{z^2 - 2} dz$ integral, where C $\left\{ z : z = 2e^{i\theta}, 0 \le \theta \le \frac{\pi}{3} \right\}$.

b) If $i^{\alpha + i\beta} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$.

SECTION-C

- 5. Take a single valued branch Of the function $F(z) = \sqrt{\cos z}$ and find its Taylor expansion about z = 0.
- 6. State and Prove Roche's Theorem.
- 7. Show that a bilinear transformation transforms two inverse points with respect to a circle or line onto inverse points with respect to the image circle or image line.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.