Roll No. □□□□□□□□□□□□ **Total No. of Pages : 02**

Total No. of Questions : 09

M.Sc. (Mathematics) (Sem.–1)

ALGEBRA-I

Subject Code : MSM-101-22

M.Code : 92796

Date of Examination : 14-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Write short notes on :

   a) Prove that a group of prime order is abelian.

   b) Show that the only abelian simple groups are groups of prime order.

   c) Suppose $a \in G$ has only two conjugates in G then show that N(a) is a normal subgroup of G.

   d) State Fundamental Theorem of finite abelian groups.

   e) Show that there are at most three groups of order 21.

   f) State Sylow's third theorem.

   g) Let G be a group of order 12. Show that either Sylow 3-subgroup is normal or $G \cong A_4$.

   h) Find all the composition series of Z_{30} and show they are equivalent.

   i) Prove that a division ring is a simple ring.

   j) Let $< Z, +, . >$ be the ring of integers. Then E = set of even integers is an ideal of Z.

SECTION-B

2. a) Let G be a finite group such that every non-identity element of G has same order n. Show that n is prime.

   b) If a cyclic subgroup K of G is normal in G, then show that every subgroup of K is normal in G.

3. a) Prove that a homomorphism $f : G \to G'$ is one-one iff $\text{K erf} = \{e\}$.

   b) Prove that any finite cyclic group of order n is isomorphic to Z_n the group of integers addition modulo n.

4. a) Prove that an abelian group G has a composition series iff G is finite.

   b) Prove that every cyclic group is solvable.

5. a) Show that a finite p-group is solvable, where p is prime.

   b) Prove that any finite group G (with at least two elements) has a maximal normal subgroup.

SECTION-C

6. a) Let H, K be two distinct maximal normal subgroups of G, then show that $G = HK$ and $H \cap K$ is a maximal normal subgroup of H as well as K.

   b) Show that all Sylow p-subgroups of G are isomorphic.

7. State and prove Sylow's first theorem.

8. a) Let R be an integral domain with unity such that R has finite number of ideals. Show that R is a field.

   b) Let $f : R \to R'$ be a ring homomorphism, then show that K erf is an ideal of ring R.

9. Let R be a commutative ring with unity, then show that an ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.