Roll No.   **Total No. of Pages : 02**

Total No. of Questions : 07

M.Sc.(Mathematics) (2018 Batch) (Sem.-1)

ALGEBRA-I

Subject Code : MSM-101-18

M.Code : 75129

Date of Examination : 17-01-2023

Time : 3 Hrs.   **Max. Marks : 70**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION - B & C. have THREE questions each.**
3. **Attempt any FOUR questions from SECTION B & C carrying FIFTEEN marks each.**
4. **Select atleast TWO questions from SECTION - B & C each.**

SECTION-A

1. **Write short answers :**

a) Find the inverse of a if $(\mathbb{Z}, *)$ is a group with $a * b = a + b - 1$; $\forall a, b \in \mathbb{Z}$?

b) Prove that there is no simple group of order 56.

c) Give an example to show that in a commutative ring R with unity, a prime ideal need not be the maximal ideal.

d) State first Sylow theorem.

e) What is a solvable group and give one example.

SECTION-B

2. a) Show that in a group of even order, the number of elements of order 2 is odd.

b) Show that a non-abelian group of order 6 is isomorphic to the symmetric group S_3.

3. a) Prove that a finite group is solvable if and only if its composition factors are cyclic groups of prime order.

b) Give an example of a non-abelian group each of whose subgroups is normal.

4. a) Prove that the alternating group A_n is simple if n > 4.

b) What is a simple group and give one example.

SECTION-C

5. a) Prove that every group of order p^2 is abelian, where p is a prime.

b) For any ring R and any maximal ideal A $\neq$ R, prove that the quotient ring R/A has no non-trivial ideals.

6. a) Prove that the sum of all the nil ideals in a ring R is itself-a nil ideal and it is the largest nil ideal in the ring R.

b) Let G be a finite abelian group of order n. Then, if p is a prime dividing n, show that there is a element $g \in$ G of order p.

7. a) State and prove second Sylow theorem.

b) Find all the homomorphisms from the ring of integers $\mathbb{Z}$ to $\mathbb{Z}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.