Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐☐   Total No. of Pages : 02

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B.Tech. (AI&ML / AI&ML and Data Science / CSE / Cyber Security / IOT / Data Science / Internet of Things and Cyber Security including Block Chain Technology) (Sem–3)

MATHEMATICS-III

Subject Code : BTAM304-18
M.Code : 76438
Date of Examination : 01-06-2023

Time : 3 Hrs.   Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :
1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION-A

1. **Write Briefly :**

    a) Examine for saddle points $f(x,y) = xy$.

    b) Change order of integration for $\int_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} f(x,y)\,dxdy$.

    c) Prove that convergent sequence has a unique limit.

    d) Discuss convergence of $\sum \dfrac{2n^2 - 2}{2^n + 1}$.

    e) State Cauchy integral test.

    f) Prove that if $M(x, y)dx + N(x, y)dy = 0$ is exact then $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

    g) Define Bernoulli's equation.

    h) Solve $(D^3 + D)y = 0$.

i) Define Legendre's differential equation

j) Solve $(x-1)^2 y'' - (x-1)y' + y = 0$.

SECTION-B

2. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2} \right)^2 u = -9(x + y + z)^{-2}$.

3. Test for convergence the series $\sum \dfrac{n!}{(n+1)^n} x^n$.

4. Discuss uniform convergence of $\sum \dfrac{a^n x^n}{n^2 + 1}$.

5. Solve $(xy^3 + y)dx + 2(x^2 y^2 + x + y^4)dy = 0$.

6. Using the method of variation of parameters, solve $\dfrac{d^2 y}{dx^2} + y = \tan x$.

SECTION-C

7. Find the volume bounded by the paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$.

8. Solve $\dfrac{dy}{dx} - \dfrac{dx}{dy} = \dfrac{x}{y} - \dfrac{y}{x}$.

9. Solve $\dfrac{d^2 y}{dx^2} + \dfrac{1}{x}\dfrac{dy}{dx} = 12\dfrac{\log x}{x^2}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.