Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ Total No. of Pages : 02

Total No. of Questions : 09

B.Tech. (Agri. Engg./Auto Engg./CE/CSE/ECE/ME/R&AI) (Sem–2)

MATHEMATICS-II

Subject Code : BTAM-203-18

M.Code : 91959

Date of Examination : 02-06-2023

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

l. Solve :

a) For the differential equation $xdy + 2ydx - xydy$, check whether the equation is exact or not.

b) Find the general solution of the Clairaut's equation $y = xy' - e^{2y'}$.

c) Find a general solution of the differential equation $y'' + y' - 2y = 0$.

d) Find the general solution of the homogeneous differential equation $x^2y'' + xy' - 4y = 0$, where $x > 0$.

e) Find the regular and singular points of the differential equation :

$(1 - x^2)y'' - 2xy' + n(n + 1)\,y = 0$.

f) Find $\lim_{z\to1} \dfrac{z^2-1}{z-1}$.

g) Show that if $f(z)$ is analytic and R$e f(z)$ = constant, then $f(z)$ is a constant.

h) Determine all the points (if any) at which the Cauchy-Riemann equations are satisfied for the function f(z) = z(Im z).

i) Evaluate $\int_a^b \phi(t)\, dt$, where $\phi(t) = t + it^2$, a = 0, b = 1.

j) State Cauchy-Goursat Theorem.

SECTION-B

2. a) Find the integrating factor and hence solve the differential equation $(x^3 + y^3 + 1)\,dx + xy^2 dy = 0$.

   b) Find the solution of the Bernoulli equation $yy' = 2x - y^2$.

3. a) Find the general solution of the differential equation:

   $x^2y'' - 2y = 2x + 6$, where $x > 0$.

   b) Solve $y = 2p + 3p^2$, where $p = \frac{dy}{dx}$.

4. Find the power series solutions about the origin of the second order equation $(1 + x^2)y'' - 9y = 0$.

5. Find the general solution of the differential equation $y'' + y = \operatorname{cosec} x$, using the method of variation of parameters.

SECTION-C

6. a) Show that the limit: $\lim_{z \to 0} \frac{z}{|z|}$ do not exist.

   b) Examine the continuity of the function $f(z) = \begin{cases} \frac{z^2+1}{z+i} & ,z \neq -i, \\ 0 & ,z = -i \end{cases}$ at $z = -i$.

7. a) Show that the function $v(x,y) = e^x \sin y$ is harmonic. Find its conjugate harmonic.

   b) Under the mapping $w = f(z) = z^2$, find the image of the region bounded by the lines $x = 1$, $y = 1$, and $x + y = 1$. Is the mapping conformal?

8. a) Evaluate the integral $\oint_C \frac{e^z}{z+1}\,dz, C: \left|z + \frac{1}{2}\right| = 1$.

   b) Expand the function $f(z) = 1/z$ about $z = 2$ in Taylor's series.

9. a) Compute the residues at the singular points of f{z), where $f(z) = \frac{z}{(z+1)(z-2)}$.

   b) Obtain the first three terms of the Laurent series expansion of the function:

   $f(z) =$ about the point $z = 0$ valid in the region $0 < |\mathbf{z}| < 2\pi$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.