

Roll No.

Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (Sem.-2)
MATHEMATICS-II
Subject Code : BTAM-203-18
M.Code : 91959
Date of Examination : 23-01-2023

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Write short notes on :
 - a) Solve $p = \log (px - y)$
 - b) Find Integrating factor of $(x^2y^2 + xy + 1) ydx + (x^2y^2 - xy + 1) xdy = 0$.
 - c) Find non-ordinary (singular) points of equation $(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + m(m+1)y = 0$, m is any real or complex number.
 - d) Solve : $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$.
 - e) Solve $\frac{dy}{dx} + \frac{y}{x} = e^x$. ($x > 0$)
 - f) For what values of z , the function $w = u + iv$ defined by $z = e^{-v} (\cos u + i \sin u)$ ceases to be analytic.
 - g) Prove that the function $v = e^{-x} (x \cos y + y \sin y)$ is harmonic.
 - h) Prove that the image of the straight line $y = mx$ is an equiangular spiral under the transformation $w = e^z$.

i) Evaluate $\int_C (3z^2 + 2z + 1) dz$, where C is the arc of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$, between $\theta = 0$ to $\theta = 2\pi$.

j) State Cauchy-Goursat Theorem.

SECTION-B

2. a) Determine for what values of a and b , the differential equation $(y + x^3) dx + (ax + by^3) dy = 0$ is exact.

b) Solve : $x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 20y = (x+1)^2$.

3. a) Solve : $x^2 \left(\frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} + 2y^2 - x^2 = 0$.

b) Solve : $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$.

4. a) Solve : $y - 2px = \tan^{-1}(xp^2)$.

b) Solve : $x \frac{dy}{dx} + y = x^3 y^6$.

5. Solve in power series, the differential equation $(1 - x^2) y'' - 2xy' + 6y = 0$.

SECTION-C

6. a) Find the analytic region of $f(z) = (x - y)^2 + 2i(x + y)$.

b) Evaluate $\int_C \frac{z+1}{z^4 - 4z^3 + 4z^2} dz$, where C is $|z - 2 - i| = 2$.

7. a) Prove that $\tan^{-1} \frac{y}{x}$ is harmonic.

b) Find the Taylor series to represent the function $\frac{z^2 - 1}{(z+2)(z+3)}$ in $|z| < 2$.

8. a) Find the analytic function $f(z) = u + iv$, whose real part u is $\frac{x}{x^2 + y^2}$.
- b) Evaluate $\int_C z^2 e^{\frac{1}{z}} dz$, where C is $|z| = 1$.
9. Find the Laurent's series expansion of $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ in the region $|z-1| > 4$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.