Roll No. | | | | | | | | | | | | Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (AE/A&R/CSE/EEE/IT/ME/CE/ME/ECE/EE) (Sem–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Date of Examination : 15-06-2023

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Write briefly :

a) What is maximum value of function $f(x) = \sin x + \cos x$

b) Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{\sin(x \cos x)}{\cos(x \sin x)}$.

c) Find the equation of tangent plane to the surface $xyz = 6$ at (1,2,3).

d) Show that the function $f(x,y) = \begin{cases} 2x^2 + y; & (x,y) \neq (1,2) \\ 0; & (x,y) = (1,2) \end{cases}$ is discontinuous at (1, 2).

e) Calculate approximate value of $\sqrt{24}$ to two decimal places by Taylor's theorem.

f) Evaluate $\int_0^1 \int_1^2 (x+5) dy dx$

g) Examine the nature of the series 1 + 2 + 3 + - - - - n - - - - - ∞

h) Define skew symmetric matrices with example.

i) Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

j) Find the eigen value of the matrix A = $\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$.

SECTION-B

2. a) Verify Rolle's Theorem for $f(x) = \sqrt{4 - x^2}$ in [–2, 2].

b) Apply Taylor's theorem with Lagrange's remainder to function $f(x) = \sin x$ in $\left[\frac{\pi}{2}, x\right]$.

3. Discuss the convergence of the following improper integral

(a) $\int_0^\infty \frac{1}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)} dx, a \neq b$ (b) $\int_2^3 \frac{x+1}{\sqrt{x-2}} dx$

4. Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

5. a) Evaluate by changing the order of integration of $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx$.

b) Find the volume common to the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinder $x^2 + y^2 = ay$.

SECTION-C

6. a) Discuss the convergence or divergence of the series $\sum \frac{\sqrt{x+n-1}}{\sqrt{x^2 + n^2 + 1}}$.

b) Test the convergence of $\sum \frac{(1+nx)^n}{n^n}$.

7. a) Test the convergence of $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+----------\infty$

b) Test the convergence of $\sum\frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$.

8. Find the value of λ for which the equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$, $(\lambda - 1)^* + (4\lambda - 2)y + (\lambda + 3)z = 0$, $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$ are consistent and find the ratios of x, y, z when A has the smallest of these values. What happens when λ has the greatest of these values.

9. a) Find a matrix B which transforms $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ into a diagonal form.

b) Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 2 & 3 & 6 \\ 0 & -1 & -3 & 1 & 1 \\ -2 & 4 & -3 & -6 & 11 \end{bmatrix}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.