Roll No. | | | | | | | | | | | | | Total No. of Pages : 03

Total No. of Questions : 18

B.Tech. (Software Engg.) (Sem.–1)

ENRICHED CALCULUS-I

Subject Code : MA-1130

M.Code : 77255

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. If $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - x - 6$. Find *gof*.

2. State second derivative test for local extrema.

3. Evaluate $\lim_{x \to 100} \frac{x-100}{\sqrt{x}-10}$.

4. Suppose $\phi(x) = \begin{cases} 3x-k, & x \le 2 \\ x+2, & x>2 \end{cases}$. Determine a value of the constant k for which $\lim_{x \to 2} \phi(x)$ exists and state the value of the limit, if possible.

5. Discuss the continuity of $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$ at $x = 3$.

6. Using the definition of derivative, find an equation of the line tangent to the graph of $f(x) = x^3 + 4x$ at (1, 5)

7. Compute the derivative of $\frac{\log x + e^x}{\sin 3x}$

8. If $x^2 + 2xy + y^3 = 42$, find $\frac{dy}{dx}$.

9. Give an example of a function that is one-to-one on the entire real number line.

10. A stone is launched vertically upward from a bridge 80 ft above the ground at a speed of 64 ft/s. Its height above the ground t seconds after the launch is given by $f(t) = -6t^2 + 64t + 80$, for $0 \le t \le 5$. When does the stone reach its maximum height?

SECTION-B

11. a) Find the inverse of the function $f(x) = \frac{1}{4}x + 1$ and write it in the form $y = f^{-1}(x)$. Also verify the relationships $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

    b) Find the domain and range of the function $f(x) = \sqrt{x^2 - 2x - 3}$.

12. a) Let $f(x) = \frac{|x|}{x}$, for $x \neq 0$. Does $\lim_{x \to 0} f(x)$ exist? Explain your reasoning after first examining $\lim_{x \to 0^-} f(x)$, $\lim_{x \to 0^+} f(x)$.

    b) Evaluate $\lim_{x \to -\infty} \left(5 + \frac{100}{x} + \frac{\sin^4 x^3}{x^2}\right)$

13 a) The side of an equilateral triangle is increasing at the rate of 2 cm/second. At what rate is its area increasing when the side of the triangle is 20 cm?

    b) Detemine the intervals of continuity for $f(x) = \begin{cases} x^2 + 1 & \text{if } x \le 0 \\ 3x + 5 & \text{if } x > 0 \end{cases}$

14. a) Find the derivative of $\frac{x^2 \sin x}{1 + \cos x}$

    b) Find the maximum value of $(x - 1)^2 + 3$ in the interval $[-3, 1]$.

SECTION-C

15. a) If $f(x) = x^{\sin x}$. Find $f'(x)$ (3)

b) A swimming pool is 50 m long and 20 m wide. Its depth decreases linearly along the length from 3 m to 1 m. It is initially empty and is filled at the rate of 1 m^3/minute. How fast is the water level rising 250 minutes after the filling begins? How long will it take to fill the pool? (5)

16. A 10-ft-tall fence runs parallel to the wall of a house at a distance of 4 ft. Find the length of the shortest ladder that extends from ground to the house without touching the fence. Assume that the vertical wall of the house and the horizontal ground have infinite extent.(8)

17. a) Find the antiderivative of the function $4\sqrt{x} - \frac{4}{\sqrt{x}} + 2x^2$. Check your answer by taking derivative. (4)

b) Evaluate $\lim_{x \to 0^+} \log_{\sin 2x} \sin x$. (4)

18. Use the graphing guidelines to graph the function $f(x) = \frac{x^3}{3} - 400x$. (8)

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.