B.Tech. (EE) PT   (Sem.–1)
ENGINEERING MATH-III
Subject Code : BTAM-301
M.Code : 70970

Time : 2 Hrs.   **Max. Marks : 30**

INSTRUCTIONS TO CANDIDATES :
 1.   **Attempt any FIVE question(s), each question carries 6 marks.**

1.   Use the concept of residues to evaluate $\int_0^{2\pi} \dfrac{d\theta}{(5-3\cos\theta)^2}$.

2.   Find a Fourier series to represent, $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence, show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}.$$

3.   Find $L^{-1}\left[\log\left(1+\dfrac{1}{s^2}\right)\right]$, where L^{-1} denotes inverse Laplace transform.

4.   If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - e^y - e^{-y}}$, find the analytic function $f(z)$, given $f\left(\dfrac{\pi}{2}\right) = 0$.

5.   Solve the equation by method of separation of variables.

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \text{ given } u = 3e^{-y} - e^{-5y} \text{ when } x = 0.$$

6.   Using the recurrence relation $(n + 1)P_{n+1}(x) = (2n + 1)\,xP_n(x) - nP_{n-1}(x)$. Hence, evaluate $P_2(1.5)$ and $P_3(2.1)$.

7. A tightly stretched string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$ the string is given a shape defined by $f(x) = \mu x\,(l - x)$, where μ is constant and then released. Find the displacement of any point x of the string at any time $t > 0$.

8. Find the power series solution about the origin of the equation :

$$\left(1 - x^2\right)y'' - 4xy' + 2y = 0.$$