Roll No. | | | | | | | | | | | | **Total No. of Pages: 02**

Total No. of Questions: 07

B.Sc. (Computer Science) (Sem. – 6)

REAL ANALYSIS

Subject Code: BCS-601

M Code: 72781

Date of Examination : 09-01-2023

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1. Write briefly:

   a) Show that the sequence $\{f_n(x)\}$, where $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ is uniformaly convergent on $[0,2\pi]$.

   b) Show that the series $\sum_1^\infty \frac{1}{1+n^2x}$ converges uniformly in $[1,\infty)$.

   c) Find the radius of convergence of the power series $\sum_{n=1}^\infty \frac{\angle(n-1)}{n^n}x^n$.

   d) Show that the series $\sum_1^\infty \frac{\sin nx}{n^3}$ is term by term differentiable.

   e) State M_n-Test for uniform convergence of sequence of functions.

   f) Show that the function $f(z) = \frac{xy^3}{x^2+y^6}, z \neq 0, f(0) = 0$ is not continuous at origin.

   g) Let R be the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ in z-plane.Determine the region R^* of w-plane into which R is mapped under the transformation$w = 3z$.

   h) Find the modulus and argument of $\left(\frac{2+i}{3-i}\right)^2$.

   i) Find the Fourier series for the function $f(x) = |x|, -\pi < x < \pi$.

   j) Write $f(z) = z^3 + z + 1$ in the form $f(z) = u(x,y) + iv\,(x,y)$.

SECTION-B

2. a) Test the sequence $\{f_n(x)\}$, where $f_n(x) = \frac{nx}{1+n^3x^2}$ for uniform convergence on $[0,1]$.

   b) Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in any interval $[0, b], b > 0$.

3. a) Prove that 0 is a point of non-uniform convergence of the series $\sum_1^\infty \frac{x}{[(n-1)x+1][nx+1]}$.

   b) Show that the series $\sum_1^\infty \frac{1}{n^3+n^4x^2}$ is term by term differentiable.

4. a) Show that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$, for $|x| \leq 1$

   b) Determine the radius of convergence of $\sum_1^\infty \angle(2n-1)x^n$.

5. a) Find an analytic function whose imaginary part is $e^{2x}(x\cos 2y - y\sin 2y)$.

   b) Show that $w = \sinh z$ satisfies the CR-equations.

6. a) Show that the function $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not differential at $z = 0$

   b) Determine the regions of z-plane for which $|z-1| + |z+1| \leq 4$.

7. a) Obtain Fourier series of the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in interval $0 < x < 2\pi$.
   b) Find the image of the infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.