Roll No. | | | | | | | | | | | | | **Total No. of Pages: 02**

Total No. of Questions: 07

Bachelor of Science (Computer Science) (Sem. – 6)

LINEAR ALGEBRA

Subject Code: BCS-602

M Code: 72782

Date of Examination : 03-01-2023

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1. Write briefly:

   a) Give the definition of Groups.

   b) Let V be the set of all real valued continuous functions defined on $[0,1]$ such that $f\left(\frac{2}{3}\right) = 2$.Show that V is not a vector space over R (reals) under addition and scalar multiplication defined by $(f+g)(x) = f(x) + g(x), (af)(x) = af(x)$, where a is any real number.

   c) Show that the intersection of two subspaces of a vector space is also a subspace.

   d) Find the value of k if the vectors $(1,-1,3), (1,2,-3)$ and $(k,0,1)$ are linearly dependent.

   e) Examine whether $(1,-3,5)$ belongs to the linear space generated by S, where $S = \{(1,2,1), (1,1,-1), (4,5,-2)\}$ or not?

   f) Prove that the polynomials $1, 2-x, 3+x^2, 4-x^3$ span the subspace W of all polynomials over reals and of degree ≤ 3.

   g) Find out that the mapping $T: R^3 \to R^2$, defined by$\mathrm{T(x,y,z)} = (|\mathrm{x}|,0)$ is a linear transform or not.

   h) Find a linear transformation $T: R^3 \to R^3$ whose Range is generated by $(1,0,-1)$ and $(1,2,2)$.

   i) Show that the linear operator T on R^3 is invertible and find a formula for T^{-1}, where $\mathrm{T(x,y,z)} = (\mathrm{x} - 3\mathrm{y} - 2\mathrm{z}, \quad \mathrm{x} - 4\mathrm{z}, \quad \mathrm{z})$.

   j) Let $T: R^2 \to R^2$ be defined by $\mathrm{T(x,y)} = (4\mathrm{x} - 2\mathrm{y}, \quad 2\mathrm{x} + \mathrm{y})$ then find matrix of T relative to the basis $\mathrm{B} = \{(1,1), (-1,0)\}$.

SECTION-B

2. a) Show that a non-empty subset W of a vector space $V(F)$ is a vector subspace of V if and only if W is closed under vector addition and scalar multiplication.

   b) If W_1 and W_2 are subspaces of a vector space $V(F)$. Then prove that $\mathrm{W}_1 + \mathrm{W}_2 = \{\mathrm{w}_1 + \mathrm{w}_2 \mid w_1 \in W_1, w_2 \in W_2\}$ is a subspace of V(F).

3. State and prove Existence Theorem for basis of a finite dimensional vector space.

4. a) Find the basis and dimension of the subspace W of R^4, generated by $(1,-4,-2,1),(1,-3,-1,2),(3,-8,-2,7)$. Also extend the basis of W to a basis of the whole space R^4.

   b) Let W_1 and W_2 be the subspaces of R^4 generated by $\{(1,1,0,-1),\ (1,2,3,0),(2,3,3,-1)\}$ and $\{(1,2,2,-2),(2,3,2,-3),(1,3,4,-3)\}$, respectively. Find the dimension of $W_1 + W_2$.

5. For the given linear transformation $T: R^3 \to R^3$ defined by$\mathrm{T(x,y,z)} = (3\mathrm{x},\quad \mathrm{x}-\mathrm{y},\quad 2\mathrm{x}+\mathrm{y}+\mathrm{z})$, find the basis and dimension of its Range and its null space. Also verify the Rank-Nullity Theorem.

6. Show that two finite dimensional vector spaces U(F) and V(F) over the same field F are isomorphic if and only if they have same dimensions.

7. Let T be the linear operator on R^3 defined by

   $\mathrm{T(x,y,z)} = (2\mathrm{y}+\mathrm{z},\quad \mathrm{x}-4\mathrm{y},\quad 3\mathrm{x})$. Find the matrix of T in the basis

   $B = \{(1,1,1),(1,1,0),(1,0,0)\}$. Also verify that

$$[\mathrm{T:B}][\mathrm{v:B}] = [\mathrm{T(v):B}].$$

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.