Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 09

B.Sc. Honours (Mathematics) (Sem.–5)

REAL ANALYSIS-II

Subject Code : UC-BSHM-501-19

M.Code : 91059

Date of Examination : 12-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Answer briefly:

a) Define the following terms in a metric space (X,d):

i) Open set

ii) Interior point

iii) Limit point

iv) Boundary point

b) State Balzano Weierstrass property

c) Prove that in a metric space (X, d) the limit of a sequence is always unique.

d) Prove that continuous image of the compact set is compact.

e) Prove that a subset A of a metric space (X, d) is open iff $A = A^{o}$

f) Define complete metric space. Give an example

g) State comparison tests of integrals for convergence and divergence of the integrals.

h) What are the improper integrals. Give examples of various types of improper integrals.

i) Define upper and lower Riemann sums. Explain when both the sums are equal.

j) Test for the convergence of the improper integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

SECTION-B

2. Define metric space and show that $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ is a metric on $\mathbb{R}^2$.

3. Let (X, d) be a metric space and let $A \subseteq X$. Then A is closed set iff $A' \subseteq A$ *i.e*, A contains all its limit points.

4. Define continuous mapping in a metric space. Let (X,d_1) and (Y,d_2) be two metric spaces and let $f : X \to Y$ be a mapping. Then f is continuous iff inverse image under f of every open subset of Y is open subset of X.

5. Define Cauchy Sequence. Show that every convergent sequence in a metric space is a Cauchy sequence.

SECTION-C

6. Prove that if a metric space is compact, then every infinite subset of X has a limit point in X.

7. Prove that closed subset of compact set in a metric space is compact.

8. Prove that intervals and only intervals are connected in a usual metric space ($\mathbb{R}$, d).

9. Test for the convergence of the improper integral $\int_{-1}^1 \frac{x-1}{x^{\frac{5}{3}}} dx$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.