Roll No. | | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 09

B.Sc. Honours (Mathematics) (Sem.–5)

PARTIAL DIFFERENTIAL EQUATIONS

Subject Code : UC-BSHM-504-19

M.Code : 91062

Date of Examination : 19-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Write short notes on :

a) What do you mean by Linear Partial Differential Equation? Give an example.

b) Form the partial differential equation by eliminating a and b from $z = \text{axe}^{y} + \frac{1}{2}a^2e^{2y} + b.$

c) Find the differential equation of all spheres having centres on x-axis.

d) Define order and degree of Partial differential equation.

e) Form the partial differential equation by eliminating arbitrary function from the equation $z = f(y/x)$

f) Solve $yz\,p + zx\,q = xy$, where $p = \frac{\partial z}{\partial x}$ & $q = \frac{\partial z}{\partial y}$.

g) Write down the Laplace equation in cylindrical coordinates (r, θ, z).

h) Classify the partial differential equation: $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$, $x \neq 0$.

i) Write down two dimensional heat equation in polar form (r, θ).

j) Write down the Wave equation in Spherical polar coordinates (r, θ, ϕ).

SECTION-B

2. Find the complete integral of $(x^2 + y^2)\,(p^2 + q^2) = 1$, $\left(\text{where } p = \frac{\partial z}{\partial x} \;\&\; q = \frac{\partial z}{\partial y}\right)$

3. Find the integral surface of $(x - y)p + (y - x - z)q = z$, $\left(\text{where } p = \frac{\partial z}{\partial x} \;\&\; q = \frac{\partial z}{\partial y}\right)$ which passes through the circe $z = 1$, $x^2 + y^2 = 1$.

4. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions:

$$u(0, y) = u(l, y) = 0;\ 0 \le y \le m$$

$$\text{and } u(x, m) = 0,\ u(x,0) = F(x);\ 0 \le x \le l$$

5. Convert the Laplace equation from Cartesian form to polar form (r, θ)

SECTION-C

6. Solve the two-dimensional wave equation in Cartesian coordinates by variable separation method.

7. A tightly stretched unit square membrane starts vibrating from rest and initial displacement is $\lambda \sin 2\pi x \sin \pi y$. Find deflection at any instant.

8. Find the temperature in a bar of length 10 cm that is perfectly insulated and heat flow in the bar is given by $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$. The ends of bar are kept at temperature 0ºc and initial temperature is $f(x) = 10x - x^2$.

9. A rod of length/insulated sides, is initially at a uniform temperature u_0. Its ends are suddenly cooled at 0°c and are kept at that temperature. Find the temperature $u(x, t)$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.