Roll No. | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 09

B.Sc- Honours (Mathematics) (Sem.–5)

ALGEBRA-II

Subject Code : UC-BSHM-502-19

M.Code : 91060

Date of Examination : 14-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Write briefly :

a) In a group G, show that $O(a) = 0(a^{-1})\ \forall\ a \in G$.

b) Write down all the elements of the permutation group S_3 on three elements 1, 2 and 3.

c) Show that center of a group is its subgroup.

d) State Lagrange's theorem.

e) Let H and K be two subgroups of group G, if H is a normal subgroup of G, then show that HK = KH is a subgroup of G.

f) State third fundamental theorem of isomorphism.

g) S_1 and S_2 are two subrings of a ring R. Then justify $S_1 \cup S_2$ is not a subring of R.

h) Show that the sum of two nilpotent elements of a commutative ring is also nilpotent.

i) Prove that identity element in a ring is unique.

j) If H be a normal subgroup of a group G and index [G : H) = m, then show, that for any $x \in G$, $x^m \in H$.

SECTION-B

2. a) Prove that the centralizer C(H) of a subgroup H in G, is a subgroup of G.

   b) Show that the product of two even permutations is again an even permutation.

3. a) Let a $\in$G be an element of group G and O(a) = m. Then show that

$$O(a^k) = \frac{m}{\gcd(m,k)}, \text{ where } k \in \mathbb{N}.$$

   b) Let a, b $\in$ G be two elements of group G, then show that O(ab) = O(ba).

4. Prove that every subgroup of a cyclic group is cyclic.

5. a) Find the center of the Quaternion group under multiplication.

   b) A non-empty subset H of a group G is a subgroup, then show that HH = H.

SECTION-C

6. a) Prove that every quotient group of a cyclic group is cyclic.

   b) Let N_1 and N_2 be two normal subgroups of a group G, prove that $G/N_1 = G/N_2$ if and only if $N_1 = N_2$.

7. State and Prove Cayley's theorem.

8. a) Prove that the sum of two ideals is also an ideal.

   b) Prove that the cartesian product of two subring of a ring is also a subring.

9. For any positive integer n, the ring $\mathbb{Z}_n$ of all integers modulo n is an integral domain if and only if n is prime.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.