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Total No. of Questions : 09

B.Sc. (Non-Medical) (Sem.–4)

LINEAR ALGEBRA

Subject Code : BSNM-406-18

M.Code : 77684

Date of Examination : 24-12-22

Time : 3 Hrs. **Max. Marks : 50**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying ONE mark each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) Check the consistency of the system of equations: $x + 2y + 3z = 0$; $2x + 3y - 2z = 0$; $4x + 7y + 4z = 0$.

b) State Cayley-Hamilton theorem.

c) Determine the eigen values of the matrix $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$.

d) Determine the row rank of the matrix $\begin{pmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{pmatrix}$.

e) If λ is an eigenvalue of the matrix A then prove that λ^k is an eigenvalue of A^k.

f) Let V be a vector space in $\mathbb{R}^3$. Examine whether $W = \{(a, b, c) : a^2 + b^2 + c^2 \le 1\}$ is a subspace of V.

g) Prove that intersection of two subspaces is also a subspace.

h) Let V be a vector space over a field F. Show that any subset S of V containing zero vector is Linearly Dependent over F.

i) State rank-nullity theorem.

j) Let $T : V \to V$ be a Linear Transformation. Show that $T(O) = O$, where O is the zero element of vector space V.

SECTION-B

2. Write the vector $\upsilon = (1, -2, 5)$ as a linear combination of the vectors $\upsilon_1 = (1, 1, 1)$, $\upsilon_2 = (2, -1, 1)$ and $\upsilon_3 = (1, 2, 3)$ in vector space $V_3(R)$.

3. Examine whether the set of vectors $\{(1, 0, -1), (1, 2, 1), (0, -3, 2)\}$ in $V_3(\mathbb{R})$ forms a basis or not.

4. Find a Linear transformation $T : \mathbb{R}_3 \to \mathbb{R}_4$ whose range is spanned by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$.

5. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 3 & -2 \\ 3 & 1 & -1 \end{pmatrix}$. Find A^{-1}. Find the eigenvalues of A and A^2.

6. Solve the system of equations: $x - y + 3z = 3$; $2x + 3y + z = 2$; $3x + 2y + 4z = 5$.

SECTION-C

7. Let V be a vector space and $T : V \to V$ is a linear transformation. Show that following statements are equivalent

   a) The intersection of range of T and null space of T is the zero subspace of T.

   b) If $T(T(\upsilon)) = 0$. Then $T(\upsilon) = 0$ for $\upsilon \in V$.

8. Diagonalize the matrix $\begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & -3 & 0 \end{pmatrix}$.

9. a) Prove that the non-homogeneous system of equations $AX = B$, where A is $m \times n$ matrix, has a solution if and only if the matrix A and the augmented matrix $(A|B)$ have the same rank.

   b) Prove that the Linear Span L(S) of any subset S of a vector space V(F) is a subspace of V(F).

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.