Roll No. [ ] Total No. of Pages : 02

Total No. of Questions : 09

B.Sc. (Non-Medical) (Sem.–4)

ANALYSIS-II

Subject Code : BSNM-405-18

M.Code : 77683

Date of Examination : 22-12-22

Time : 3 Hrs. **Max. Marks : 50**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying ONE mark each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) State Cauchy's criterion for uniform convergence of sequence of functions.

b) Show that 0 is a point of non-uniform convergence of the sequence $\{f_n(x)\}$ when $f_n(x) = nxe - nx^2,\ x \in \mathbb{R}$.

c) State Weirstrass M-test.

d) Show that $\sum \frac{\cos n\theta}{nP}$ is uniformly convergent for all real values of θ, where $p > 1$.

e) Find the divergence of function $f(x, y, z) = xy^2\hat{i} + 2x^2yz\hat{j} - 3yz^2\hat{k}$ at the point $(1, -1, 1)$.

f) Determine the constant 'a' so that the vector $\vec{V} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x-az)\hat{k}$ is solenoidal.

g) State Gauss's divergence theorem.

h) Find b_n in the Fourier series expansion of the function $f(x) = |x|$ in the interval $(-\pi, \pi)$

i) State Weirstrass approximation theorem.

j) Find a_0 in the Fourier series expansion of the function $f(x)$ in the interval $(-\pi, \pi)$, where

$$f(x) = \begin{cases} 0 & -\pi < x \le 0 \\ \dfrac{\pi x}{4} & 0 < x < \pi \end{cases}$$

SECTION-B

2. Expand $f(x) = \pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$ up to the first three terms.

3. Evaluate $\iiint_V \phi \, dV$, where $\phi = 45x^2y$ and V is the closed region bounded by the planes $4x + 2y + z = 8$, $x = 0$, $y = 0$, and $z = 0$

4. Prove that div $(r^n \vec{r}) = (n+3)\, \vec{r^n}$.

5. Show by M_n test, 0 is a point of non-uniform convergence of the sequence $\{f_n\{x\}\}$ where $f_n(x) = 1 - (1 - x^2)^n$.

6. Test for uniform convergence and continuity of the sum function of the series for which $f_n(x) = \dfrac{1}{1+nx}$, $0 \le x \le 1$.

SECTION-C

7. a) Given the series: $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} - ....$, show that one can obtain the expansion of $\cos x$ by differentiation.

   b) Examine for term by term integration for the series whose sum of first n terms is $n^2 x (1 - x)^n$.

8. a) Find the Fourier series for the function $f(x) = x^3$ in $(-\pi, \pi)$

   b) Evaluate $\oint_C (x^2 - \cosh y)\, dx + (y + \sin x)\, dy$ by Green's theorem, where C is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $(\pi, 1)$, and $(0, 1)$.

9. If $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ find the value of $\iint_S \vec{F}.\hat{n}\, dS$, where S is the closed surface bounded by the planes $z = 0$, $z = 1$, and the cylinder $x^2 + y^2 = 4$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.