Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 03**

Total No. of Questions : 09

B.Sc. (Mathematics) (Sem.–4)

LINEAR ALGEBRA

Subject Code : UC-BSHM-403-19

M.Code : 79912

Date of Examination : 22-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Write short notes on :

a) Let V be a vector space in $\mathbb{R}^3$. Examine W = $\{(a, b, c) \mid c$ is an integer$\}$ is a subspace or not.

b) If S and T are any subsets of a vector space V (F). Prove that $S \subset T \Rightarrow L(S) \subset L(T)$.

c) Let V be a vector space over a field F. Then prove that every non zero singleton subset of V is L.I. over F.

d) Give examples of two different basis of V_3 (R).

e) Examine the mapping $T : R^2 \to R$ defined by $T(x, y) = xy$ is linear transformation or not.

f) Find the rank of zero transformation.

g) Find a linear transformation $T : R^3 \to R^4$ whose range is spanned by (1, 2, 0, –4) and (2, 0, –1, –3).

h) Show that there is no non-singular linear transformation from R^4 to R^3.

i) If λ is an eigenvalue of the matrix A then prove that $\lambda \pm k$ is an eigenvalue of $A \pm kI$.

j) Let T : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a L.T. s.t. T $(x, y) = (2x + 5y, 6x + y)$. Compute minimal polynomial of T.

SECTION–B

2. a) Prove that the union of two subspaces is a subspace if and only if one of them is a subset of the other.

   b) For what value of k, will the vector $\upsilon = (1, k, -4) \in R_3$ is a linear combination of $\upsilon_1 = (1, -3, 2)$ and $\upsilon_2 = (2, -1, 1)$.

3. a) Let V and W be two vector spaces over the same field F and T : V $\rightarrow$ W be a linear transformation with kernel K. prove that

   i) K is a subspace of V

   ii) T (V) is a subspace of W.

   b) Let V (F) be a vector and T, S be two linear transformations on V. Then show that if T and S are invertible, then TS is also invertible.

4. a) Let T be a linear operator on R^3 defined by T $(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}.

   b) For the linear transformation T : $R^2 \rightarrow R^3$, defined by, T $(x, y) = (x + y, x - y, y)$, find a basis and dimension of (i) its range space (ii) its null space.

5. a) Let S and T are any subsets of V (F). Prove that L (S $\cup$ T) = L (S) + L (T).

   b) Show that the set $B = \{1, x, x^2, x^3, \ldots., x^n\}$ of $(m + 1)$ polynomials is a basis set for the vector space $P_m(R)$ of all polynomials of degree m over R (reals).

SECTION-C

6. a) Let V (F) and W (F) be two vector spaces and T : V $\rightarrow$ W is a linear transformation. Assume that V (F) is finite dimensional. Prove that V and range space of T have the same dimensions iff T is non-singular.

   b) Let T be a linear operator on R^3 defined by T $(x, y) = (2y + z, x - 4y, 3x)$

      i) Find the matrix of T relative to the basis B = {(1, 1, 1), (1, 1, 0), (1, 0, 0)

      ii) Verify that [T ; B] [υ ; B] = [T (υ(; B] $\forall\, \upsilon \in R^3$.

7. a) Show that L.T., $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x - y, y - z, z - x)$ is neither one-one nor onto.

b) Find the matrix representation of linear transformation $T : R^2 \rightarrow R^3$ defined by $T(x, y) = (x + 4y, 2x + 3y, 3x - 5y)$ relative to ordered basis $B_1 = \{(1, 1), (2, 3)\}$ for R^2 and $B_2 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ for R^3.

8. Diagonalize the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

9. a) Find the characteristics roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify the Cayley-Hamilton theorem for this matrix. Find A^{-1} and also, express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A.

b) Express the quadratic from $2x^2 + 5y^2 - 6z^2 - 2xy - yz + 8zx$ in matrix notation.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.