Roll No. [| | | | | | | | | | |] **Total No. of Pages : 03**

Total No. of Questions : 09

B.Sc. Hons. (Mathematics) (Sem.–4)

VECTOR CALCULUS

Subject Code : UC-BSHM-401-19

M.Code : 79910

Date of Examination : 17-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Write short notes on :

a) $\vec{a}, \vec{b}$ and $\vec{c}$ are vector functions of a scalar function t then what is the value of $\frac{d}{dt}[\vec{a} \times (\vec{b} \times \vec{c})]$.

b) If $\hat{R}$ is a unit vector in the direction of $\vec{r}$, prove that $\hat{R} \times \frac{d\hat{R}}{dt} = \frac{\vec{r}}{r^2} \times \frac{d\vec{r}}{dt}$, where $r = |\vec{r}|$.

c) What do you mean by directional derivative?

d) Show that a vector field given by $\vec{A} \times (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$ is irrotational.

e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that div $\vec{r} = 3$.

f) If $u\vec{F} = \nabla v$, where u, v are scalar fields and $\vec{F}$ is a vector field, show that $\vec{F}.curl\,\vec{F} = 0$.

g) Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at the points $t = \pm 1$.

h) State Gauss divergence theorem.

i) If $\vec{F} = 3xy\hat{i} - y^2\hat{j}$, evaluate $\int_C \vec{F}.d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from (0, 0) to (1, 2).

j) What do you mean by surface integral?

SECTION – B

2. Find the unit tangent vector at any point on the curve $x = t^2 + 2$, $y = 4t - 5$, $z = 2t^2 - 6t$, where t is any variable. Also determine the unit tangent vector at the point $t = 2$.

3. The position vector of a moving particle at a time t is $\vec{R}(t) = t^2\hat{i} - t^3\hat{j} + t^4\hat{k}$. Find the tangential and normal components of its acceleration at time $t = 1$.

4. Prove that four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar if and only if $[\vec{b}\ \vec{c}\ \vec{d}] + [\vec{c}\ \vec{a}\ \vec{d}] + [\vec{a}\ \vec{b}\ \vec{d}] = [\vec{a}\ \vec{b}\ \vec{c}]$.

5. Given $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$, find the reciprocal triads $\vec{a}', \vec{b}', \vec{c}'$ and verify that $[\vec{a}\ \vec{b}\ \vec{c}][\vec{a}'\ \vec{b}'\ \vec{c}'] = 1$.

SECTION-C

6. Find the value of constants a, b and c so that the maximum value of the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, –1) has a magnitude 64 in the direction parallel to z-axis.

7. Evaluate $\int_C (yzdx + (zx+1)dy + xy\,dz)$, where C is a straight line joining the points (1, 0, 0) to (2, 1, 4).

8. Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2 dy]$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

9. State and prove Stoke's theorem.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.