Roll No. | | | | | | | | | | | | **Total No. of Pages: 02**

Total No. of Questions: 07

B.Sc. (CS) (Sem. – 4)

NUMBER THEORY

Subject Code: BCS-401

M Code: 72317

Date of Examination: 13-12-2022

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1. Write briefly:

   a) How many integers between 100 and 1000 are divisible by 7?

   b) Prove that there are no pair of integers x, y satisfying $x + y = 100$ and $(x, y) = 3$.

   c) Using Euclidean Algorithm, find L.C.M. of 306 and 657.

   d) Prove that every integer of the form $8^n + 1$ is composite for $n \geq 1$.

   e) Show that there are no integers x such that $x \equiv 29 (\text{mod } 48)$, $x \equiv 11 (\text{mod } 50)$, $x \equiv 72 (\text{mod } 135)$

   f) Find the remainder when 15 ! is divided by 17 .

   g) Find the value of $\phi(360)$.

   h) Find the highest power of 9 dividing 365!.

   i) Find the number of positive integers ≤ 3600 that are prime to 3600.

   j) Find the missing digit x of $51840 \cdot 273581 = 1418243\, x\, 040$, using working modulo 9 or 11.

SECTION-B

2\. a) Find all solutions of $91x + 221y = 1053$, if they exist.

b) Prove that every integer n>1 can be represented as a product of primefactors in only one way, apart from the order of factors.

3\. a) Find values of x and y to satisfy $71x - 50y = 1$.

b) Find all integers a and b satisfying $(a, b) = 10$ and $[a, b] = 100$.

4\. a) Solve the given linear congruence $13x \equiv 3(\text{mod}47)$.

b) Find the remainder when 2^{50} is divided by 7.

5\. a) Prove that for any positive integer n, $\sum_{d=1}^{n} \phi(d) \left[\frac{n}{d}\right] = \frac{n(n+1)}{2}$.

b) Let F and f are two arithmetic functions. For every integer n, if

$F(n) = \sum_{\frac{d}{n}} f(d)$ then $f(n) = \sum_{\frac{d}{n}} \mu(d) F\left(\frac{n}{d}\right)$.

6\. a) Using Fermat's theorem, if p is an odd prime, then show that

$$1^p + 2^p + \cdots . + (p-1)^p \equiv 0(\text{mod } p).$$

b) Find the least non-negative residue of $(583)^{361}(\text{mod } 91)$.

7\. a) Solve $x \equiv 2(\text{mod } 3), x \equiv 3(\text{mod } 5), x \equiv 5(\text{mod } 2)$.

b) If p is a prime then show that

$$(p-1)! \equiv p - 1(\text{mod } 1 + 2 + 3 + \cdots + (p-1)).$$

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.