Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Sc. (Non-Medical) (Sem.–3)

ANALYSIS-I

Subject Code : BSNM-305-18

M.Code : 76904

Date of Examination : 02-01-23

Time : 3 Hrs. **Max. Marks : 50**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying ONE marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) State Gauss test

b) State Raabe's Test

c) State fundamental theorem of Integral Calculus.

d) Define lower and upper Riemann integral.

e) Give an example of a Riemann integrable function on [a, b] which is not monotonic [a, b]

f) Compute $\Gamma\left(-\frac{1}{2}\right)$.

g) Explain Beta function.

h) State Dirichlet's test.

i) Expain absolute convergence of improper integrals of second kind.

j) Prove that $\sum \frac{n}{n^2+2}$ is divergent.

SECTION-B

2. Test the convergence of $1+\frac{(1+\alpha)}{(1+\beta)}+\frac{(1+\alpha)(1+2\alpha)}{(1+\beta)(1+2\beta)}+\frac{(1+\alpha)\,(1+2\alpha)(1+3\alpha)}{(1+\beta)(1+2\beta)(1+3\beta)}+...$

3. Test for the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{3!}+....\infty$.

4. Prove that if f is monotonic in [a, b], then it is integrable in [a, b],

5. Show that $\int_0^{\infty}\frac{x}{1+x^4\sin^2 x}$ is divergent.

6. Prove that $\int_0^1(1-x^n)^{\frac{1}{n}}\,dx=\frac{\left(\Gamma\left(\frac{1}{n}\right)\right)^2}{2n\Gamma\left(\frac{2}{n}\right)}$.

SECTION-C

7. a) If f is Reimann integrable function on [a, b], Show that the function F defined by F(x) $\int_a^x f(t)\,dt\ \forall\, x\in[a,b]$ is uniformly continuous.

   b) Prove that the function f defined by

   $$f(x)=\frac{1}{2^n}\text{ when }\frac{1}{2^{n+1}}\le x\le\frac{1}{2^n},\quad (n=0,1,2,\dots\ \dots\ \dots)$$

   Is integrable on [0, 1], although it has an infinite number of discontinuities. Also show that $\int_0^1 f(x)\,dx=\frac{2}{3}$.

8. If $\int_0^{\infty} f(x)\,dx$ is convergent at ∞ and $\phi\ (x)$ is bounded and monotonic for $x\ge a$, then $\int_0^{\infty} f(x)\phi\ (x)\,dx$ is convergent at ∞.

9. Show that $\int_0^1\frac{x^2dx}{\sqrt{1-x^4}}\times\int_0^1\frac{dx}{\sqrt{1+x^4}}=\frac{\pi}{4\sqrt{2}}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.