Roll No. | | | | | | | | | | | | **Total No. of Pages : 03**

Total No. of Questions : 09

B.Sc. Honours (Mathematics) (Sem.–3)

REAL ANALYSIS-I

Subject Code : UC-BSHM-302-19

M.Code : 78497

Date of Examination : 14-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

I. Write short notes on :

a) Define bounded above and bounded below sets.

b) Find the infimum and supremum of the set $\left\{\frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$.

c) State the completeness property of $\mathbb{R}$.

d) Define convergent sequence. Also give an example.

e) Define Cauchy sequence and also give an example of a sequence which is not a Cauchy sequence.

f) Using n^{th} term test, show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+....+\frac{n}{n+1}+...$ is not convergent.

g) State Cauchy root test for infinite series.

h) Define absolutely and conditionally convergent series.

i) Define alternating infinite series.

j) Give an example of absolutely convergent series.

SECTION-B

2. a) Show that $\sqrt{2}$ is not a rational number.

   b) For all real numbers x, y show that $|x+y| \le |x| + |y|$.

3. Which of the following sets are bounded above, bounded below, bounded or unbounded? Also find the supremum (1.u.b.) and infimum (g.l.b.), if they exist. Which of these belong to the set?

   a) $S = \left\{-1, \frac{-1}{2}, \frac{-1}{3}, ...\right\}$

   b) $(a, b]$

   c) $\left\{\frac{4n+3}{n}, n \in \mathbb{N}\right\}$

   d) The set of all real numbers.

4. a) Show that every convergent sequence is bounded but converse of this theorem need not be true.

   b) If a sequence $\{s_n\}$ converges to l, then every subsequence of $\{s_n\}$ converges to l. Also show that converse of this result is not true.

5. a) Show that $\left\{\frac{1}{n}\right\}$ is a Cauchy sequence.

   b) Prove that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is monotonically increasing, bounded and tends to the limit $\frac{2}{3}$.

SECTION-C

6. a) By using the concept of sequence of partial sums, show that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges.

b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$ converges if $p > 1$ and diverges if $p \le 1$.

7. a) By using the ratio test, test the convergence of the following series $\sum \frac{n!}{n^n}$.

b) Test the convergence of the series $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-n^2}$.

8. a) Show that the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ is convergent.

b) Test for absolute convergence the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{(n+1)!}$.

9. a) Using the Abel's test, show that the series $\sum \frac{(n^3+1)^{\frac{1}{3}} n}{\log n}$ is convergent.

b) Test for convergence and absolute convergence of the series.

$$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{7}} + ...$$

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.