Roll No. | | | | | | | | | | | | | **Total No. of Pages : 03**

Total No. of Questions : 09

B.Sc. Honours (Mathematics) (Sem.–3)

CALCULUS-III

Subject Code : UC-BSHM-301-19

M.Code : 78496

Date of Examination : 12-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Write short notes on :

a) State geometrical meaning of partial derivatives of first order.

b) Check for continuity :

$$f(x,y) = \begin{cases} 2x^2 + y\,; & (x,y) \neq (1,2) \\ 0\,; & (x,y) = (1,2) \end{cases} \text{ at } (1,2)$$

c) Define Homogeneous functions with the help of example.

d) If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, verify $\dfrac{\partial(x,y,z)}{\partial(r,\theta,z)}$.

e) If $u = \sin^{-1}(x-y)$, $x = 3t$, $y = 4t^3$, show that $\dfrac{\partial u}{\partial t} = \dfrac{3}{\sqrt{1-t^2}}$.

f) What do you mean by error in Taylor's series?

g) State necessary condition for a function $f(x, y)$ to have an extremum at (a, b).

h) State any two advantages of Lagrange method.

i) Evaluate $\int_0^1 \int_{x^2}^{2-x} x\,y\,dydx$.

j) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\cos\theta} \int_0^{\sqrt{a^2-r^2}} r\,dz\,drd\theta$.

SECTION-B

2. If $x = r\cos\theta$, $y = r\sin\theta$ prove that :

a) $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial x^2} = 0$ b) $\frac{\partial^2 r}{\partial x^2} \cdot \frac{\partial^2 r}{\partial y^2} = \left(\frac{\partial^2 r}{\partial x \partial y}\right)^2$.

3. Compute f_{xy} (0, 0) and f_{yx} (0, 0) for the function :

$$f(x,y) = \begin{cases} \frac{xy^3}{x+y^2}; & (x,y) \neq (0,0) \\ 0; & (x,y) = (0,0) \end{cases}$$

Also, discuss the continuity of f_{xy} and f_{yx} at (0, 0).

4. If $u = x + y^2$, $v = y + z^2$, $w = z + x^2$, then prove that $\frac{\partial x}{\partial u} = \frac{1}{1+8xyz}$. Also, find $\frac{\partial^2 x}{\partial u^2}$.

5. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

SECTION-C

6. State and Prove Taylor's theorem for a function of two variable. Using it expand e^{xy} at (1, 1).

7. Examine the function $x^3 + y^3 - 3axy$ for maxima and minima.

8. Prove that $\iiint_V \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\, dxdydz = \frac{\pi^2}{4} abc$.

9. Find the volume bounded by the xy plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.