Roll No. | | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 07

B.Sc. (CS) (Sem.–3)

SOLID GEOMETRY

Subject Code : BCS-301

M.Code : 71773

Date of Examination:12-12-2022

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students has to attempt any FOUR questions.**

SECTION-A

1. Write briefly :

a) Reduce the equation $x^2 + 4y^2 + 3z^2 + 2x - 8y + 9z - 10 = 0$ into the standard form and identify the quadratic surface represented by it.

b) If a right circular cone has three mutually perpendicular generators, then show that its vertical angle is $\tan^{-1}\sqrt{2}$.

c) Shift the origin to a suitable point so that the equation $2x^2 + 3y^2 + z^2 + xy + zx - 10y - 4z + 22 = 0$ is transformed into an equation in which the first degree terms are absent.

d) Explain the section of a sphere by a plane.

e) State the condition for three planes to intersect in a point.

f) Prove that the angle between the lines given by

$x + y + z = 0$, $ayz + bzx + cxy = 0$ is $\frac{\Pi}{2}$ if $a + b + c = 0$

g) Identify the following surface: $16z^2 - 4x^2 - 8z + 8x - 3 = 0$.

h) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 0$ and find the point of contact.

i) Find the equation of the sphere passing through (1,0,0), (0,1,0), (0,0,1) and whose centre lies on the plane $3x - y + z = 2$.

j) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 = 49$, $2x + 3y + 6z = 14$.

SECTION-B

2. a) Find the equation of the sphere circumscribing the tetrahedron whose faces are $x = 0$, $y = 0$, $z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

b) Find the locus of the centres of the spheres passing through the fixed point (0, 2, 0) and touching the plane $y = 0$.

3. a) A variable plane is parallel to the given plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axis in A, B, C. Prove that the circle ABC lies on the cone $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$

b) Find the equation of cone whose vertex is at (–1, 1, 2) and whose guiding curve is $3x^2 - y^2 = 1$, $z = 0$.

4. a) Find the centre of the two spheres which touch the plane $4x + 3y = 47$ at the point (8, 5, 4) and the sphere $x^2 + y^2 + z^2 = 1$.

b) Find the equations of the two tangent planes to the sphere $x^2 + y^2 + z^2 = y$, which pass through the line $x + y = 6$, $x - 2z = 3$.

5. a) Find the equation of the cylinder whose generator are parallel to the line $\frac{x}{4} = \frac{y-4}{5} = \frac{z+1}{-4}$ and which has for its guiding the curve hyperbola $3x^2 - 4y^2 = 5$, $z = 2$.

b) Obtain the equation of the right circular cylinder describe on the circle through the points (a,0,0), (0,a,0) and (0,0,a) as the guiding circle.

6. a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and find the point of contact.

b) Find the equation of the tangent planes to sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which pass through the lines $x + z - 16 = 0$, $2y - 3z + 30 = 0$.

7. a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 0$ and find the point of contact.

b) Transform the equation $13x^2 + 13y^2 + 10z^2 + 8xy - 4yz - 4zx - 144 = 0$ when the axes are rotated to the axes having direction cosines $\left\langle -\frac{1}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle, \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle$ and $\left\langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right\rangle$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.