Roll No. | | | | | | | | | | | | **Total No. of Pages: 02**

Total No. of Questions: 07

B.Sc (CS) (Sem. – 3)

SEQUENCE SERIES AND CALCULUS

Subject Code: BCS-302

M Code: 71774

Date of Examination: 14-12-2022

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1. Write briefly:

a) Show that the sequence $\{a_n\}$, where $a_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots . + \frac{1}{n\cdot(n+1)}$ is convergent.

b) If a sequence is convergent then it converges to a unique limit.

c) Show that the series $\frac{1}{2^3} - \frac{1+2}{3^3} + \frac{1+2+3}{4^3} - \frac{1+2+3+4}{5^3} + \cdots$ is convergent

d) Test the convergence or divergence of the series $\sum \frac{(n+1)^n}{n^{n+1}} x^n$.

e) Discuss the convergence of the series $\sum \frac{1}{3n-1}$.

f) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\ \theta} d\theta = \frac{\pi}{\sqrt{2}}$.

g) Show that the function $f(x) = \begin{cases} 0, \text{ when } x \text{ is rational} \\ 1, \text{ when } x \text{ is irrational} \end{cases}$ is not Riemann integrable on any interval.

h) Show that $3 \leq \int_3^4 (7x^2 + 3x)dx - \int_2^3 (2x^3 + 5x)dx \leq 98$.

i) Examine the convergence of $\int_e^{\infty} \frac{dx}{x(\log\ x)^{\frac{3}{2}}}$.

j) Examine the convergence of $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$.

SECTION-B

2. a) If $\{a_n\}$ sequence of positive terms and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists whether finite or infinite then show that $\lim_{n\to\infty} (a_n)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

   b) Show that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is monotonically increasing, bounded and has limit $\frac{2}{3}$.

3. a) Test the convergence or divergence of the series $\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \cdots, x > 0.$

   b) Show that the series $\sum \frac{n^n x^n}{\angle n}$.

4. a) Show that the series $\sum(-1)^{n-1}\frac{x^n}{n}$ is convergent for $-1 < x \leq 1$.

   b) Discuss the convergence of the series $\sum \frac{1^2\cdot 3^2\cdot 5^2 \ldots.(2n-1)^2}{2^2\cdot 4^2\cdot 6^2 \ldots\ldots.(2n)^2} x^{n-1}$.

5. a) Show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

   b) Prove that $\int_0^{\infty} \frac{x^{m-1}}{(a+bx)^{m+n}}\,dx = \frac{1}{a^n\cdot b^m}\beta(m,n)$, where a, b, m, n are all positive.

6. a) Show that the necessary and sufficient for a bounded function f to be R-integrable on $[a,b]$ is that to every $\in > 0$, however small, there exists a partition P such that $U(P,f) - L(P,f) < \in$.

   b) By considering the integral $\int_n^{n+1} \frac{1}{x}dx, n > 0$, prove that $\frac{1}{n+1} \leq \log\left(1+\frac{1}{n}\right) \leq \frac{1}{n}$

7. a) Compute $L(P,f)$ and $U(P,f)$ for the function $f(x) = \sin x$, where $P = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$.

   b) Discuss the convergence of the integral $\int_2^3 \frac{dx}{(x-2)^{\frac{1}{4}}(3-x)^2}$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.