Roll No. | | | | | | | | | | | | | **Total No. of Pages : 02**

Total No. of Questions : 09

B.Sc. (Hons.) Mathematics (Sem.–2)

CALCULUS - II

Subject Code : UC-BSHM-201-19

M.Code : 77765

Date of Examination : 20-12-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. a) Prove that the curve $y = e^x$ is concave upwards for all $x \in \Upsilon$.

   b) Find the point of inflexion of the curve $y = x^4$.

   c) Find the asymptotes parallel to the coordinate axes of the curve $x^2 + y^2 = 4x^2 + 9y^2$.

   d) Find the envelope of the family of straight lines $y = mx + \frac{1}{m}$. where m is the parameter.

   e) Examine the nature of origin on the curve $y^2 (a^2 + x^2) = x^2 (a^2 – x^2)$.

   f) If $I_n = \int (\log x)^n dx,$ then slow that $I_n + I_{n-1} = x(\log x)^n$.

   g) Test the convergence of the integral $\int_1^{\infty} \frac{\log x}{x^2} dx$.

   h) Using fundamental theorem of calculus, find $c \in [0, 6]$ such that $f(c)$ equals the average value of the function $f(x) = \frac{x}{2}$ over $[0,6]$.

   i) Find the length of the arc of the parabola $y^2 – 4y + 2x = 0$ which lies in the first quadrant.

   j) Give the formula for Simpson's $\frac{1}{3}$ rule.

SECTION-B

2. a) Find the intervals in which the curve $y = (\cos x + \sin x)\, e^x$ is concave upward or downward in $(0, 2\pi)$.

   b) Determine a and b so that the curve $y = ax^3 + bx^2$ has a point of inflexion at $(-1, 2)$.

3. Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$

4. Trace the curve $y = x^3 + 5x^2 + 3x - 4$.

5. a) Prove that the curvature of a straight line is zero.

   b) Find the evolute of the parabola $y^2 = 4ax$.

SECTION-C

6. Obtain a reduction formula for $\int x^n e^{-x} dx$. Hence evaluate $\int_0^\infty x^n e^{-x} dx$, where n is a positive integer.

7. Show that the integral $\int_0^\infty \sin(x^2) dx$ is convergent.

8. a) Find the area bounded by the lines $y = x$, $x = -1$ and $x = 1$.

   b) Use the midpoint rule to estimate $\int_0^1 x^2 dx$ using four subintervals. Compare the result with the actual value of this integral.

9. Determine the surface area of the solid obtained by rotating $y = \sqrt{9 - x^2};\ -2 \le x \le 2$ about the x –axis.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.