Roll No. [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ] **Total No. of Pages: 02**

Total No. of Questions: 07

B.Sc. (CS) (Sem. – 2)

PARTIAL DIFFERENTIATION & DIFFERENTIAL EQUATIONS

Subject Code: BCS-201

M Code: 71506

Date of Examination : 20-12-2022

Time: 3 Hrs. **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1. Write briefly:

a) If $z = f(x + ct) + \phi(x - ct)$, then prove that $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$

b) Let $x = r\cos\theta\,, y = r\sin\,\theta$, show that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$.

c) State Euler's theorem for homogeneous functions of three variables.

d) Find $\frac{dw}{dt}$ at $t = 0$, where $w = xy + yz + zx, x = t^2, y = te^t,\ z = te^{-t}$.

e) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right), 0 < x, y < 1$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot\,u$

f) Define exact differential equation. Give condition such that given differential equation is exact.

g) Define integrating factor. Is integrating factor unique for a given differential equation? Justify with a suitable example.

h) Write standard form of Clairaut's equation and find general solution of $p = \sin\,(y - xp)$.

i) Solve $(D^3 - D^2)y = 0$.

j) Show that family of curves given by $y^2 = 4a(x + a)$ are self-orthogonal.

SECTION-B

2. Show that the function

$$f(x,y) = \begin{cases} \dfrac{x^2+y^2}{x-y}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is not continuous at (0,0) but its partial derivatives f_x, f_y exist at (0,0).

3. If $u = f(r)$ and $x = r\cos\theta\,, y = r\sin\,\theta$, then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r^2} f'(r).$$

4. If $\tan\,u = \frac{x^3+y^3}{x-y}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin\,2u$ and

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin\,u)\sin\,2u.$

5. Solve $y = 2px + y^2p^3$.

6. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = \tan\,2x$.

7. Find a series solution of the equation $2x^2y'' + xy' - (x^2+1)y = 0$ about the point $x = 0$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.