Roll No. $\square\square\square\square\square\square\square\square\square\square\square\square$ **Total No. of Pages : 03**

Total No. of Questions : 09

Bachelor of Science - Honours (Mathematics) (Sem.–1)

CALCULUS-I

Subject Code : BSHM-101-22

M.Code : 92788

Date of Exmination : 21-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. Write short notes on :

a) Give the domain of the function $\sqrt{x^2-1}$.

b) Express $y=\sqrt{\sin x}$ as composite of appropriate functions.

c) Give $\in - \delta$ definition of continuity of a function. Explain the difference between continuity and the uniform continuity of the function.

d) Express $y=\sin\sqrt{x^2+1}$ as composite of appropriate functions

e) Prove that the function

$$f(x)=\begin{cases} x, & x\leq 1 \\ 2-x, & 1<x\leq 2 \\ -2+3x-x^2, & x>2 \end{cases}$$

is continuous at $x = 1$ and $x = 2$.

f) Give the domain of the function $\sqrt{1-|x|}$.

g) Find the n[th] derivative of $y = \cos^4 x$

h) State Cauchy's mean value theorem.

i) Using first Principle differentiate $y = \log x$.

j) Define One-One function and Onto function. Give an example of each.

SECTION-B

2. Examine the discontinuity of the function

$$f(x) = \begin{cases} \dfrac{|x-a|}{x-a} & x \neq a \\ 1 & x = a \end{cases}$$

at point $x = a$.

3. a) Differentiate $\sin^{-1}\left(\dfrac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}\right)$

b) Find the derivative of $y = \tan x \,.\, \tan h\, x$.

4. a) Using L'Hospital's rule find limit of

$$\frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$$

where $x \to 0$.

b) Explain the difference between first and second kind of discontinuity.

5. a) If $y = \sqrt{\sin x \sqrt{\sin x \sqrt{\sin x + ...\infty}}}$, find $\dfrac{dy}{dx}$.

b) A particle moves along a straight line such that 's' is a quadratic function of 't', prove that its acceleration remains constant.

SECTION-C

6. State and prove Cauchy's mean value theorem. Show that Langrange's mean value theorem is particular case of Cauchy's mean value theorem.

7. a) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $(e)^{\frac{1}{e}}$

   b) Explain graphically the meaning of $\frac{dy}{dx} > 0$ and $\frac{dy}{dx} < 0$.

8. a) If $x + y = 1$, then prove that $\frac{d^n}{dx^n}(x^n y^n) = n![y^n - ({}^nC_1)^2 y^{n-1} \;.x({}^nC_2)^2 y^{n-2} x^2$ $+ ...(-1)^n x^n]$.

   b) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)\, y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

9. a) Obtain the expansion of log(1 + x) in powers of x by Maclaurin's theorem.

   b) Apply Taylor's theorem to expand log{sin(x + h)) in powers of h.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.