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Total No. of Questions : 09

Bachelor of Science - Honours (Mathematics) (Sem.–1)
CALCULUS-I
Subject Code : UC-BSHM-101-19
M.Code : 77312
Date of Examination : 14-01-23

Time : 3 Hrs.   Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :
1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

l. **Solve the following :**

   a) Graph the function $y = x^2$ over the interval $[-2, 2]$.

   b) If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find $(f \, o \, g)(x)$.

   c) Evaluate $\lim\limits_{x \to -2} \dfrac{3x+4}{x+5}$.

   d) Find $\dfrac{dp}{dq}$ if $p = \dfrac{1}{\sqrt{q+1}}$.

   e) Find $\dfrac{dy}{dx}$ where $y = 2u^3$, $u = 8x - 1$.

   f) Find the absolute maximum values of $f(x) = x^2$ on $[-2, 1]$.

   g) Find the intervals on which $g(x) = -x^3 + 12x + 5$, $-3 \le x \le 3$ is increasing.

   h) State Leibnitz Theorem.

   i) Find fourth derivative of $e^{ax} \sin bx$ at $x = 0$.

   j) Evaluate $\displaystyle\int_{0}^{\ln 2} 4e^x \sinh x \, dx$.

SECTION-B

2. a) Prove $\lim\limits_{x \to 1} (2x+5) = 7$ using $e - \delta$ definition.

   b) Integrate $x^3 - 6x^2 + 15\,x$ w.r.t. x.

3. a) A 13 ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/s.

   i) How fast is the top of the ladder sliding down the wall then?

   ii) At what rate is the area of the triangle formed by the ladder, wall and ground changing then?

   b) Find the asymptotes of the graph of $f(x) = \dfrac{x^2 - 3}{2x - 4}$

4. a) By differentiating $x^2 - y^2 = 1$ implicitly, show that $\dfrac{dy}{dx} = \dfrac{x}{y}$.

   b) A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Find dx/dt when $x = 3$.

5. a) If $x^3 + y^3 = 16$, find the value of $\dfrac{d^2 y}{dx^2}$ at the point (2,2).

   b) Find the derivative of $g(t) = \tan (5 - \sin 2t)$ w.r.t. t.

SECTION-C

6. a) Find the value of c of the Mean Value Theorem, if $f(x) = x^{2/3}$ in $[0, 1]$.

   b) Using logarithmic differentiation find the derivative of y with respect to x, where $y = (x + 1)^x$.

7. Using Lagrange's Mean value theorem, show that $| \cos b - \cos a | \leq | b - a |$.

8. Find the interval in which $y = 3x^4 + 4x^3 - 6x^2 + 12x + 2$ is concave upward.

9. Using Taylor's series, find the value of $\cos 31°$ correct to 3 decimal places.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.