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Total No. of Questions : 11

B.Sc. (Honours) Chemistry (Sem.-1)

MATHS-I (CALCULUS-I)

Subject Code : UC-BSHM-104-19

M.Code : 77226

Date of Examination : 19-01-23

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of EIGHT questions carrying TWO marks each.
2. SECTION-B contains EIGHT questions carrying FOUR marks each and students have to attempt any SIX questions.
3. SECTION-C will comprise of two compulsory questions with internal choice in both these questions. Each question carries TEN marks.

SECTION-A

1. Attempt the following :

a) Find $\lim\limits_{x\to 0} \frac{|x|}{x}$.

b) If $u(x, y) = \frac{x^3 + y^3}{x + y}$, $(x, y) \neq (0, 0)$, then evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

c) Find $\frac{dy}{dx}$. if $y + \sin y = \cos x$.

d) Find $\int (x^{2/3} + 2e^x - \frac{1}{x})dx$.

e) Find $\frac{\partial f}{\partial x}$, where $f(x, y) = x^2 + y^2 + x$.

f) $\int_{-1}^{1} \sin^5 x \cos^4 x \, dx$.

g) Find $\frac{df}{dt}$ at $t = 0$, where $f(x, y) = x \cos y + e^x \sin y$, $x = t^2 + \mathrm{I}$, $y = t^3 + t$.

h) Discuss the existence of limit $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$.

SECTION-B

2. Show that very differentiable function is continuous.

3. Verify Rolles theorem for the function $f(x) = x^2 + 2x - 8$, $x \in [4, 2]$.

4. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$.

5. Find $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx$.

6. If $\tan u = \frac{x^3 + y^3}{x - y}$, then verify Euler's theorem :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial^2 y} = (1 - 4\sin^2 u) \sin 2u .$$

7. Show that the variables $u = x - y + z$, $v = x + y - z$, $w = x^2 + xz - xy$ are functionally related.

8. Evaluate the integral $\iint_R e^{x^2} dxdy$, where the region R is given by: $2y \le x \le 2$ and $0 \le y \le 1$, by changing the order of integration.

9. Evaluate the integral $\iiint_T x \, dxdydz$, where the boundary of T is : $y = x^2$, $y = x + 2$, $4z = x^2 + y^2$, $z = x + 3$.

SECTION-C

10. Evaluate the integral $\iint_R \sqrt{x^2+y^2}\,dxdy$ by changing to polar coordinates, where R is the region in the xy-plane bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

OR

a) Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

b) Evaluate $\int \frac{x^2}{x^2+3x+2}dx$.

11. a) Find the extreme values of $f(x, y, z) = xyz$ subject to $x + y + z = 6$.

b) If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then show that $\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2$.

OR

a) If $f(x) = \int_0^x t\sin t\,dt$, then find $f'(x)$.

b) If $y = \sin^{-1}x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.